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WHAT IS THE IMPACT OF ICT ON MATHEMATICS EDUCATION?

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DECLARATION

I do hereby attest that I am the sole author of this project/thesis and that its contents are only the result of the readings and research I have done. Permission has been obtained from persons and institutions mentioned to include their interviews and their case studies.

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My special appreciation goes to the Almighty Jesus for the gift of life and hope. Also my special thanks to Dr. Mari Cruice, from the University of Roehampton for all her time and continuous encouragement to help me to embark on the Doctorate programme. I am thankful to Crisan Cosette at the University College of London, who really helped me to understand the role of digital technology and inspired my curiosity in wanting to explore more into the role of ICT in mathematics education. Also, my special appreciation are extended to Mr Neil Barley, Head of Mathematics Department, Lilian Bailey High School, whose lectures elicited valuable insights about the perceptions of ICT-integrated pedagogy held by most teachers. Additionally, my appreciation and gratitude are extended to my family and two boys Aaron and Moses; the thought of them made me press on to complete this work despite the hard and difficult times I went through in embarking on this research work.

I dedicate this thesis to Aaron and Moses and wish them success in their educational pursuit.

ABSTRACT

The analysis is based on a detailed investigation of primary data collected through case studies, questionnaires and semi-structured interviews in secondary schools in south London. The pedagogy of mathematics education is explored in order to understand the issues surrounding ICT integration. This study shows that despite the perceived advantages of ICT integrated pedagogy, there are barriers that are closely connected to teachers' pedagogical beliefs at work. The lack of ongoing professional training programmes, and curriculum expectations were admitted by most educators as among the factors that affect their attitudes towards ICT pedagogy. Further empirical research is suggested to advance the exploration of the role of subject leaders in ICT-integrated pedagogy. This research provides an in-depth analysis of the exploration of own practice and mathematics teachers perceptions on an integrated ICT into the teaching. The analysis on own practice was presented through the lenses of TPACK and the Knowledge Quartet. Some works that have been used in the past, such as the works of Rowland, Ruthven, and Mishra are used in this study, to provide the basis for comprehension of the discussion at hand.

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CHAPTER 1: INTRODUCTION

1.1 Introduction

This section gives an over view of the aim of the study, significance and professional relevance of embarking on this research. Also discussed here is the justification of undertaking this research through the chosen research questions. Finally, the structure of the study is subsequently revealed.

1.2 Aim of study

My appreciation in the use of ICT in lessons has made me question the lack of ICT in mathematics education. It is this frustration that has led me to pursue this empirical investigation to establish the role of ICT-integrated pedagogy in lessons through case studies, questionnaires and interviews. In the midst of falling educational standards as reported by UNESCO (2005), ICT-integrated lessons can raise attainment. Also, ICT integration pedagogy is important to facilitate a generation of critical thinking ability, decision making ability and effective communication (Valtonen et al., 2015).

The topic for the research was chosen in order to best understand the implications to enhance professional efficiency and to aid the development of mathematics teachers. Where ICT is integrated well, it has been shown to enhance pupils' achievements and understanding mathematics. Using ICT is now deemed by parents and educators as 'an integral part of providing a high-quality education" (U.S. DOE, 2003: 3).

Additionally, the central idea of this research paper is to add to the existing research through exploration on the impact of integrating ICT in Mathematics education. These aims will be achieved in the following ways;

First, I will explore the role of mathematics education and the role of ICT in education. Then, I will present a theoretical overview of teachers' pedagogical beliefs as a vital starting point. Afterwards, I aim to use own experience and views of mathematics teachers across selected schools in south London to established the impact of ICT integration in mathematics lessons and propose understanding of the implications to schools for future research.

1.3 Significance and professional relevance

The use of different ICT tools has significantly changed teaching practice in secondary schools in western countries (UNESCO, 2002), as ICT integration can make learning more interesting and easier for students (Buabeng-Andoh, 2012). The use of various ICT tools can also increase the pace of learning for all students. The knowledge of the role of ICT will in no doubt contribute to professional practice and impact on pedagogical choices. With this in mind, the research will endeavour to appreciate existing study on the topic to review own perception and significance on the impact of ICT tools on secondary mathematics pedagogy.

Koh and Chai (2014) are of the view that educational institutes in their current form are not able to meet the expectations of teachers in terms of ICT pedagogy knowledge. As a subject leader and teacher, the awareness of the benefits of ICT requires an immediate response into ways in which ICT can be embedded into teaching and learning as directed by the school curriculum.

Thus, from a professional point of view, the integration of ICT into pedagogy will facilitate the use of modern technologies to improve teaching. From my professional experience, it is evident that the use of ICT equips teachers to reduce the effort and time taken for the management of students' learning (Buabeng-Andoh, 2012). Also, the use of ICT can ease the information collection process and the overall process of lesson planning. Furthermore, the use of new technologies in ICT pedagogy likewise enables like-minded professionals to become creative and develop the ability to manage the process of acquiring knowledge (Livingstone, 2012).

1.4 Research Justification

The aim of this research is to explore the impact of integrating ICT into the pedagogy of secondary mathematics education. This will be done by bringing together the findings and the key points from available literature on ICT in Mathematics education. The initiatives related to attitudes that promote effective ICT pedagogy will be reviewed. In addition, the study will provide information on for schools to integrate ICT pedagogy in mathematics education. Finally, it will offer insight into the differences and exclusive characteristics of the learning needs of practitioners.

1.5 Research Questions

The effective application of ICT in education is influenced by the perceptions of teachers (Betz and Fassinger, 2011). Teachers' perceptions have become vital in using ICT tools in an era in which modern technologies are used to advance pedagogies in schools (UNESCO, 2005). The research questions were developed to explore into detail on the main research question, 'what is the impact of ICT use in mathematics education?' Although, Higgins et al., (2005) states that the benefit of teaching through ICT enhances learning, however, commenting on the obstacles of ICT-integrated lessons, Moseley et al., (1999) cited in Higgins (2005) lament the reluctance of ICT use by teachers due to the lack of ICT experience and poor attitudes. The attitudes of teachers towards ICT integration have also been associated with the perception of barriers to using ICT (Newhouse, 2001). Further, Lim and Khine (2006) argue that the problem is worsening with the absence of the required expertise by teachers to facilitate the delivery of ICT integration in schools.

The sub division of the research questions again seek to offer more insights into the entire analysis. The emerging knowledge from this study optimally is aim to enlighten schools and policy makers on the implications of using ICT in mathematics education.

The following research questions emerged for exploration in this study:

- What are the benefits of ICT in Mathematics education?
- What are the barriers to ICT in mathematics education?
- What factors will facilitate ICT in Mathematics education?

1.6 Structure of the study

This dissertation consists of five further chapters as follows: Chapter 2 presents a review of the literature on effective ICT pedagogy. Chapter 3 outlines the research methodology used and the chosen approach with its justification. Chapter 4 elaborates on the justification of the chosen data analysis method and presents findings from both the questionnaires and the semi-structured interviews. Chapter 5 provides a discussion and analysis of findings, and Chapter 6 contains the final conclusions with recommendations.

1.7 Definition of terms

- Teacher knowledge: This entails the technology put into practice by the teachers in solving problems or inventing useful tools in teaching.
- Teachers' 'use' of technology: This entails the application of technological knowledge by the teachers in education to achieve a specified goal.
- Information Technology: The use of software and computer hardware for teaching.
- Communications Technology: All Information, Communication, and Technology (ICT) equipment for communication in education.
- Implementation: The application and operation of ICT equipment in schools.
- Technology: This is the deployment of scientific and technical knowledge aimed at practical purposes, more specifically in industries and in institutions.

The literature review that follows explores publications in the field of educational technology, including research papers, journals, reports, and published databases, and draws conclusions regarding current studies on ICT integration in mathematics education.

1.8 Chapter summary

The section provided the aims of the study and the significance for undertaking the research. Furthermore, the rationales behind the research questions with the structure of the study were addressed to provide a course of the study. Also discussed briefly were the definition of terms associated with information communication and technology to aid the understanding of the research.

CHAPTER 2: LITERATURE REVIEW

2.1 Introduction

I believe that among the aims of the mathematics education is the intention to provide the skills needed to succeed on the modern job market. Morden technologies have a role to play in making this dream a reality. In this section, first, I will engage with the theory related to education and the curriculum in schools. Second, I will recognise the role of ICT within the mathematics pedagogy and finally, I will draw out any known framework aiding the effective design of an ICT in mathematics lesson with an associated technological device.

The following research questions guides this study:

- What are the benefits of ICT in Mathematics education?
- What are the barriers to ICT in mathematics education?
- What factors will facilitate ICT in Mathematics education?

2.2 Reasons for Education in schools

Historically, the vision of education existed for rulers who continued through to most of the twentieth century. After 50 years of the British comprehensive school education, family welfare still remains an important determinant of how well a child does in school (Hatcher in Cole, 2006). This sentiment of social economic factors influencing the academic progress of learners is obvious in today's educational system. The fact still remains that the greatest determinant of a school's success is the proportion of children from poor social backgrounds (Whitty, 2002). It is no surprise that studies of educational success in the UK consistently show correlations with social background, suggesting that while the aim of education can be achieved through the curriculum provided by the schools, in most cases, the effect of the education system has been producing the same knowledge over several decades (Ross, 2000).

2.2.1 Education for individual reasons

The aim of education, according to Capel et al., (2013) is seen as to provide individuals with the key processes needed to succeed in everyday life. The aim of education has changed over time and a historical perspective by Harris (1995) shows that 'education' contents are changing and becoming personalised, despite the influence of politics on the curriculum. For

Corrigan, et al., (2007), education provides the mechanism where individuals are offered the opportunities, knowledge and required skills to be a self-independent. The ultimate purpose of education however, according to Tyler (1949), is not to 'cover' particular content but should be to define attitudinal aims that ensure the opportunity of individual's progress. Reiss (2007) also indicates that education seeks to develop the benefits of individuals as well as the community. This aim of education has brought about much divide between the academic community and politicians on whether the aim of education is being fulfilled. In light of this tension, the aim of education has been used to describe all activities that define the human existence, including happiness, opportunities and the ability to make decisions (Michael and White, 2014).

2.2.2 Education for social harmony

Plato advocates the aim of education as the maintenance of social harmony through communal knowledge (Wheelahan, 2015). For Hutchins (1968) the purpose of education is to propagate the cultural truth in society and according to Rousseau (1968) the need for education is to generate social freedom to understand the meaning of life and the environment. Truly, is it not the aim of education to separate the intelligent from the less intelligent and consequently offer the intellectuals the ability to rule the less intelligent through the power offered by the curriculum? This sentiment was expressed by Bernstein (1977) when he pointed out that the prevailing aim of the education system had been to redistribute knowledge to enhance the principles of social control.

2.2.3 Education to develop a human capital

The human capital is defined by the OECD as the knowledge, skills, competencies and attributes embodied in individuals that facilitate the creation of personal, social and economic well-being (OECD, 2007, p. 29). Such development of human capital led to over half of the percentage of economic growth from the previous decades in countries such as Germany, Italy, Greece and the Netherlands in the 1980's (OECD, 2000). On a whole, each extra year of full-time education in the OECD countries corresponded to about a 10 per cent rise in human capital that was associated with about a 6 per cent increase in output per capita (OECD, 2001). The economic success of countries is based on the efficient utilization of both tangible and intangible resources, including the skilled labour force provided through the educational (Capel et al., 2013).

2.2.4 Education as a need to standardized learning

There is more progress in advocating for equal aims for all as education is too important to society and cannot be understated and restricted to the few (Leask el al., 2013). Many developments over the years to promote equal opportunity of education for all, regardless of race, sex, gender and religious affiliations, are embarked by various governments in the pursuit of eradicating the historical divisions in education. For example, in 1990 the Department of Education and Science (DES, 1978) introduced the integration of SEN children into mainstream education, after decades of separation in special education provisions (Leask el al., 2013).

The debate still continues whether the policy on equal aims for everyone in education is still possible as schools continue to embark on segregated policy in the name of appreciating the diverse needs of learners. Schools continue to promote single sex education within a mixed school, to promote achievements and segregate ability at the point of admissions. The critics are quick to conclude that such a system will promote education integration, in the midst of acknowledging the peculiarity of learners needs. Ability selection has further characteristics of endorsing segregation in the class system. Although the previous regimes criticised the segregation practices of the past, and promised to reform the equality aim of education, most schools have gone backwards in the name of promoting standards in education by acknowledging the differences in learners (Leask et al., 2013).

2.2.5 Education as a national curriculum

Among the aims of the national curriculum in England is to make children “learn and achieve; and to promote pupils’ spiritual, moral, social, cultural and emotional well-being” (DfE, 2013, p. 6). Added aims in the 2008 National Curriculum generally became the starting point of the curriculum design that ensured that all children enjoyed learning and would become successful learners. It especially directed that young people became progressive and also safe, confident and learned in a healthy-environment (DfE, 2012b). We can take a closer look at what the aim of education is by understanding the reasons for the creation of the curriculum through which the aim of education is realised. Capel et al., (2013) questioned whether all the key concepts to be gained through education could be used for why education can be captured by just the acquisition of skills. The aims of education defined how the curriculum should be shaped and formed (Reiss & White, 2014).

According to Young (2014) the curriculum should be defined as what is possible and not act as a restriction on students learning, in relation to discrete teaching of subjects and/or cross curricular teaching. Young proposed again that the current crises in curriculum theory can be overcome, not only by providing a curriculum that does not inform learners of past knowledge, but by also informing them of present and future knowledge (Young, 2013). However, if the curriculum as seen by Ross (2000) was content-based and consisted of present and future knowledge, who could deliver such content based curriculum? Since neither the content nor knowledge-based approach was considered as an appropriate medium of education (Skilbeck, 1976).

The importance of the curriculum, according to (Dewey, 1897), was not to identify it with a specific subject but should consist of the experiences of the children own social activities. In developing a curriculum Michael and White (2014) questioned whether the aim of the curriculum should guide the design of the curriculum. However, Richard Peters (1959) reiterated that the need of a curriculum cannot be sought, hence it is aimless. According to Aberdeen Standish, the notion that education is aimless makes the concept of defining a curriculum with an aim unattainable (*Standish*, 1999; Reiss, & White, 2014). The curriculum should aim to provide a medium through which the educational system can appreciate the previous knowledge acquired by learners and progress learners, with fresh knowledge to provide children with differentiated understanding including emotions support and the sense of the community (Schleicher, 2015).

2.3 The mathematics education

The mathematical education paradigm is not speculative to just a body of an isolated knowledge but are associated with the processes of enquiry into the representations of both personal and published knowledge. The mathematical philosophy on theorems and Proofs is not just the analysis of merely significant units but complex connections and collaborations between individuals, culture, social structures, unrestrained and other practices (Brown, 1995; Ernest, 1991).

This section focuses the current mathematics education in UK and then discusses the role of theorems and Proofs, to gain mathematics knowledge. It uses the example on the Pythagoras theorem to demonstrate how mathematics knowledge is perceived. The subsequent section discusses what constitutes a mathematical proof/truth and a review on the role of inductive reasoning and deductive reasoning in mathematics education, the section concludes with the use of diagrams and visualisation to gain mathematics knowledge.

2.3.1The mathematics curriculum in UK

The secondary school mathematics curriculum in England aims to develop a student's ability to theorise, inquire, reason and communicate mathematically (DFE, 2014). The mathematical knowledge provides the foundation through which the world can be understood by offering skills needed for enjoyment (DFE, 2014). It is assumed that the knowledge gained from studying the curriculum will enable young people to succeed in post-16 studies. Stephen Ball (1990) and Paul Ernest (1991) insist that a curriculum in mathematics should consist of a more traditional set of facts and skills that concentrates on ensuring that the basic knowledge on written algorithms for the primary mathematical operations are adhered to. Focusing largely on the skills as an 'industrial trainer' was commonly viewed by many prominent politicians who insisted that the mathematics curriculum should enable learners to appreciate knowing the times-tables and be able to perform sums as the core objective of the mathematical module (Brown, et al., 2000). As pointed out by Nick Gibb, a former conservative minister, the need to perform mathematical procedures and the ability to recollect the times-tables was the needed skills that the mathematics curriculum provided at the primary level.

Michael Gove (2010), the former educational secretary, viewed the ability to learn concepts in the mathematics curriculum as absurd and insisted it was insignificant to learning maths (Brown, et al., 2000). Bourdieu and Passeron (1990) did not advocate and support the initiative of an alternate curriculum, in the midst of addressing the crises in the present curriculum. Instead, they advocated that it should provide pupils with the ability to gain knowledge in transferable skills. Students' performance on mathematics has significantly impacted on a school's resources to ensure higher attainment. The government initiative to promote academic rigour came after the publication of the performance by OCED countries, which saw a decline in England's position suggesting that schools perhaps needed to promote more challenging curriculum contents. The clear absence of the government explicitly defining the expectation of 'an academic rigour' in the national curriculum, as pointed out by many reviews, suggests contention (White & Bramall, 2000).

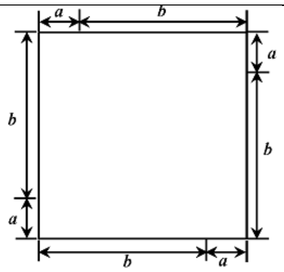
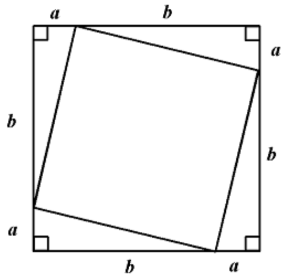
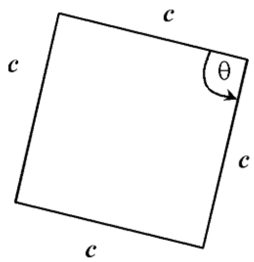
2.3.2 What is a mathematical proof/truth?

The meaning of mathematical proof has been debated by many mathematical authors over the centuries. According to Todd CadwalladerOlsker in the article *What Do We Mean by Mathematical Proof?* (2011), mathematical proofs are activities embarked on by individuals or institutions to justify a mathematical concept. The author further alleged that there is the widely held view as to whether mathematical proofs are 'discovered' or 'created'; in either case, it is people who discover or create mathematics. Hersh (1993) and others have contended with the formal meaning of whether a mathematical proof is important to the mathematics pedagogy. According to CadwalladerOlsker (2011), the issue of mathematical proofs 'continues to be negotiated and to evolve' (p. 34). Here, the notion is expressed that all mathematical propositions are not known to be true or false until they have been proved.

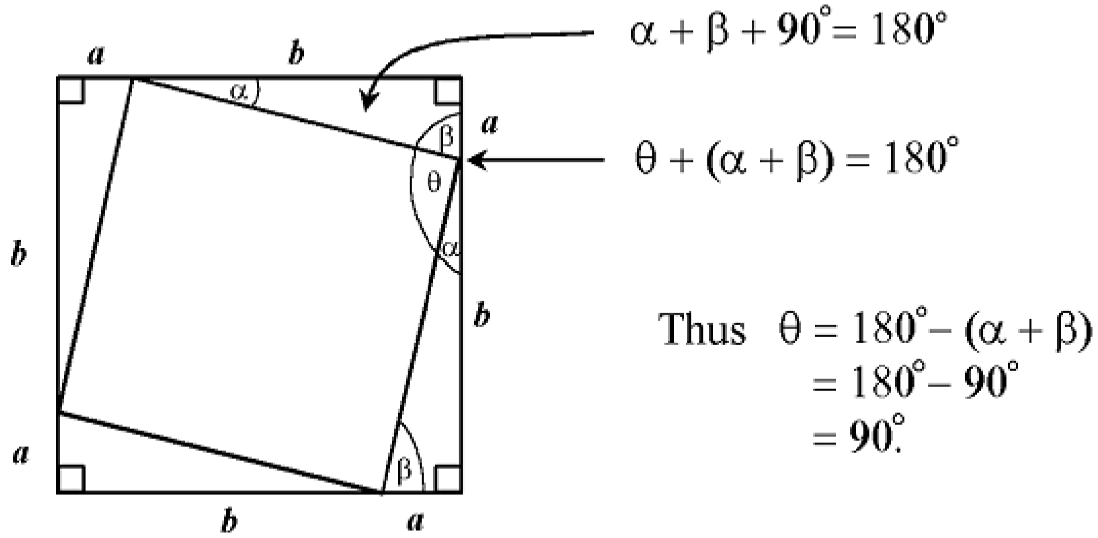
In the view of Rota, a mathematical proof involves a sequence of steps that lead to the desired mathematical conclusion. Again, he emphasises realisation that the rules and sequences of steps to be followed to achieve the desired results have not changed over the centuries (Rota, 1997). Rota agrees with what most authors indicate to be the aim of all proofs: to follow a series of logical steps to justify a mathematical fact. Nevertheless, Rota indicates that this is unrealistic in defining what most mathematicians actually think about mathematical proofs.

There are various selections involved in conjecturing for instance the Pythagorean Theorem and establishing its proof. To guide the choices that mathematicians make there are background results published in literature on the theoretical ideas, which may be detailed or may be vague. However, as time progresses various literatures are produced on the same topic and definitions of what was once an important theorem also change. For instance, while most mathematicians working on proving the Pythagorean Theorem are guided by the ideas of Euclid, some use geometry and others use the guiding principles of circles. For example, Salame et al., (2016) used geometry to prove Pythagoras' theorem, as shown below.

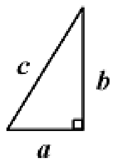
The most elementary proof of the Pythagorean Theorem can be done using a square with equal sides and lengths.

<table>
<tr>
<td>
</td>
<td>This can be extrapolated to make four right – angled triangle.

</td>
</tr>
<tr>
<td>Because all the triangles are right-angled triangles with lengths a and b, this means that their hypotenuses will have the same length c. When these triangles are removed from the larger square, the remaining shape is a quadrilateral-sided square whose sides are the triangles’ hypotenuses c.</td>
<td>
</td>
</tr>
</table>

The figure below proves that the inside angle θ of the quadrilateral is a right angle. Proportionally, all the inside angles are right angles.



Therefore, the quadrilateral forms a square with an area of c^2. The fact that the original square is 4× in the area means it is equal to the area of the quadrilateral. Which means $(a + b)^2$- 4 × area is equal to c^2. In this case



But the area of the triangle of width ***a*** and height ***b*** is $\frac{1}{2}$ *ab*. Therefore:

$(a + b) 2 - 4 \times \frac{1}{2} ab = c^2$

$a^2 + 2 ab + b^2 - 2 ab = c^2$

$a^2 + b^2 = c^2$

Mathematicians agree that proofs are an important element of learning mathematics and should be part of all students' mathematical experiences at all levels (Waring, 2000) in the early school years (Greenes, 2003). Waring (2000) and Greenes (2003) noted that students' work in the early years emphasises proof as an explanation of the structure of a known pattern. Also noted was the ability for teachers to use proof activities to advance mathematical understanding, particularly using proofs consisting of patterns to advance students' reasoning as they progress through to the higher school years. In the light of the different stages of proofs that advance the understanding of mathematics education, Waring (2000) mentioned three key distinguishers, starting from the moment of appreciation of the reason for a proof. Secondly, acknowledging their understanding of the type of proof, and finally, gaining the skills required to construct proofs (Greenes, 2003).

2.3.3 Visualisation and proofs

The ancient Greek mathematicians used proofs to demonstrate a mathematical understanding in a purely graphic method (Miller, 2012). Piaget (1936) and Van Hiele's (1986) theories of learning geometry, affirming the hierarchical stages that justifies the knowledge in mathematics and describes the stages that a learner go through in learning it. According to Van Hiele's (1986) a learner uses different stages including visualisation by children to arrive at a solution. Comparing the theorem to the proof of for example, the circle theorem, using a dynamic tool can facilitate mathematical understanding through the visualisation function that makes it possible by dragging the points on the drawn shape to align with the desired knowledge (Rodd, 2010). The act of visualisation in geometrical reasoning to represent proofs as an absolute truth is subject to debate and discussion (Giaquinto, 2007). Although visualisation offers ways of thinking about investigations in geometry, the process involves the use of heavy effects in visual reasoning that can be ambiguous for learners. Again, an individual's mood psychologically suggests that visualisation employs effects in different ways, including skills in number and algebra, difficult concepts for most learners to establish their relations with geometry. The 'positive affect' here is that when a learner is receptive they can broaden their perspective and so facilitate creative problem solving. The converse is true when the 'negative affect' contributes to a person attending to detail and so facilitates on difficult computations and indulge in efficient task accuracies (Linnenbrink & Pintrich, 2004).

2.3.4 Theorems and proofs

According to Rene Thom (1971, p. 697), 'A theorem is above all the object of a vision'. In Thom's aphorism, he defines a theorem as 'that which is true' and understanding/knowledge as the ability to be translated into certainty ('object') and revelation ('vision'). The nature of truth associated with theorems can be visually expressed (Davis, 1993). Despite the claim by many mathematicians that theorems are often viewed as the ultimate truth of a key concept in mathematics education, Thom asserts that the knowledge of a theorem can be vague and capricious.

Mathematical proofs are usually offered in a progression method, moving from one stage to the next, unidirectional from propositions to conclusion (Leron, 1983). Although this approach secures the required validity of proofs, it is, on the other hand, inappropriate when the proof involves ambiguous mathematical communication. Mathematicians in arriving at a proof have used various confusing approaches involving a process of justification and modification of ideas (Liljedahl, 2007). Communication forms a vital part in the process of any mathematical proof. In proofing, the mathematician ‘provides the reasoning and the rational underpinnings for why the mathematics behind the whole process is not only valid but also worthy of discussion’ (Liljedahl, 2007).

Many writers, including Liljedahl (2007), have indicated that the problems of learners associated with proofs must not be attributed to slow cognitive development or lack of logical reasoning power, but the inability to appreciate the need for proving theorems. According to Liljedahl (2007), proving for instance the Pythagoras’ theorem is a cognitive development of mathematical concepts.

2.3.5 Inductive reasoning

All students have the potential to participate in some form of mathematical reasoning. The nature of mathematical reasoning involved in mathematical proofs is largely categorised under either inductive or deductive reasoning. Young learners, according to Greenes (2003), indulge in inductive reasoning, which involves the ability for learners to make generalisations by looking for specific patterns. Such reasoning allows students to begin solving problems, for example through drawing pictures and using accustomed methods to make conjectures (Greenes, 2003). However, as pointed out by *Professional Standards for Teaching Mathematics* (NCTM) in 1991, although inductive reasoning aids problem solvers to develop an understanding of a problem, it may be inadequate for learners to arrive at the correct and expected solution. There is the need to verify simplifications acquired over inductive reasoning proven through deductive reasoning.

2.3.6 Deductive reasoning

Inductive reasoning depends on observing patterns and the ability to make conjectures. However, in sharp contrast, deductive reasoning depends on logic. Using proofs to solve mathematical problems enables learners to make a logical argument by drawing on logical conclusions, through generalisations based on logic applied to specific situations. For example, the proof of the Pythagorean Theorem by students may consist of an explanation through a deductive process (Greenes, 2003).

An update of the *Professional Standards for Teaching Mathematics* (NCTM, 2000) advocates teachers to use proofs to advance the mathematical reasoning of students. Using mathematical proofs to aid mathematics understanding offers learner's opportunities for developing their thinking and learning. Proofs that require students to execute a memorised procedure improve their chances of operating at a higher level of mathematical thinking, especially where the task engages learners to think actively about the concepts in the proof and make connections among them (Greenes, 2003). According to (Doyle, 1988), the tasks that students encounter through proving a mathematical theorem build their experiences and their ability to personally engage in mathematics pedagogy.

Again, the illustration demonstrated in the 1999 video study accomplished by the Third International Mathematics and Science Study (TIMSS), revealed a similar trend in the relationship between low-level tasks, where learners demonstrate mathematical procedures consisting of basic computations, and high-level tasks, which entail the making of connections centred on discovering and concepts (NCES, 2003; Stiegler & Hiebert, 2004). In an assertion, the report concluded that the tasks that promoted high-level cognitive demands on students had a maximum prospective to develop learners' ability to reason and conjecture mathematically and the ability to comprehend what defines a mathematical proof.

2.4 The benefit of ICT in education

The benefit of ICT as widely used, encompasses the view that ICT refers not only to 'computers and computing related activities' but all technological activities. The shift in definition towards the end of the 1980s from 'computers' to 'IT' represented not only computer technology, but also the integration of all related technology with the capacity to store and retrieve information (Pelgrum & Law, 2003). In this study, the term ICT is used in accordance with the United Nations report (1999) description of ICT as consisting of telecommunications equipment, information technology equipment and services, information and communication activities, internet services, and network-based information services.

2.4.1 Excitement and confidence

Children’s excitement when learning with computers allows them to devote more attention to understanding key process (Ruthven & Hennessy, 2003). Integrating ICT in learning brings a new dimension to teaching and learning, where learners go to the computer environment with the expectancy of doing something new. The use of ICT brings variety to classroom learning, which enhances learning appeal and promotes different learning styles (Oldknow & Taylor, 2003; Ruthven & Hennessy, 2003). According to Ruthven and Hennessy (2003), when the different leaning needs are accounted for while ICT lessons are introduced increases pupils’ motivational to learn new ideas. The use of ICT has the additional benefit of building confidence and persistence among pupils. The new curiosity makes learners more active and encourages them to investigate and explore new knowlwdge, so mathematical software can be relied on as a tool that improves topical understanding in mathematics, ensuring more enthusiasm to work (Ruthven et al., 2008). Again Ramsey et al., (1991) cites that students can enhance their own knowledge through enthusiasm and not remain passive recipients of information when ICT is integrated through a student-centred approach to learning.

2.4.2 Aid lesson planning with wide range of resources

The use of ICT aids teachers planning of lesson in numerous ways. According to Tinio (2002) through the use of ICT teachers gain access to online materials saving precious time and cost for creating new resources. The use of ICT enables teachers to either use or improve on either own or another teachers resources. Through the use of ICT, teachers can easily create a reservoir of variety of resources to be used by other teachers on similar topics of interest. The ability for teachers to plan to use a wide range of resources in lessons helps to accommodate for the different learning needs in the classroom when ICT is used. Through the use of the internet, teachers are able to provide learners the learning experiences beyond the classroom environment, all over the word to enhance learning (World Bank report, 2004).

The use of technology as indicated by Wong et al., (2006) is vital to support direct teaching and learning in the classroom, possible through audio and visual capabilities. The use of ICT has the powerful benefit of enabling teachers to plan different learning scenarios favourable for both collaborative and independent learning (Cabero, 2001). These learning styles are possible due to the ability of teachers to promote reception of information and interactions through the use of ICT. The use of ICT therefore, accelerates the nature of lesson delivery in mathematics education since learners can gain access to wide range of knowledgeable resources from anywhere with no geographical constraints. This benefit of using ICT to prepare these lessons resources in this approach aids the way lessons are prepared and taught, the learning is more productive, efficient and student oriented.

2.4.3 Improved the attainment of children

Many European countries, including Sweden, continue to stress the importance of integrating ICT pedagogy to enhance the quality of teaching (Abbott, 2001). Vygotsky's theory, originally written in Russian in the late 1930s, placed emphasis on the learning gained by pupils through ICT pedagogy. Where a child's learning is hampered with misunderstanding, Vygotsky (1978) emphasised the benefit of using ICT to provide guarded questions to progress the learning improved the attainment of children. Vygotsky (1978) acknowledged that children's learning of social rules and culture is enhanced by ICT-integrated learning, beneficial for educational pedegogy. Ekanayake and Wishart (2015) also argued that introducing students to ICT tools was important to help them to understand the corporate environment. Finally, the use of ICT and new technologies has increased educational attainment through independent learning (Ertmer et al., 2012). In the view of Robertson and and Al-Zahrani (2012) and Young (2014), the increase attainment is achieved because students develop more self-learning abilities through the use of ICT.

Over the last decade, digital technologies (DT) have supported the pedagogy of mathematics in schools. The impact of technology on students' mathematical thinking cannot be over exaggerated, and the debate on the effect of ICT on students' thinking and reasoning in mathematics has gained enormous attention. Olive and Makar (2010), who examined the effect of ICT on students' learning, reiterate the need to incorporate ICT to promote the efficiency of learning mathematics. According to Goos (2010), the knowledge gains in learning mathematics when digital tools are used generally outweigh the benefits of learning through pen and paper. Again Goos indicates that by using ICT, it is possible to make predictions and test conjectures when completing a mathematical task (Goos, 2010). Conferring with Zbiek et al., (2007) and Heid and Blume (2008), promoting ICT pedagogy, including the use of GeoGebra, has the possibility to support mathematical thinking to deepen students' mathematical content knowledge.

2.4.4 Enhance interactivity and exploration

Ager further claims that ICT is important to motivate and provide interactive programmes to support children's learning (Ager, 2000). The theory of Multiple Intelligences propounded by Gardner also suggests that learning takes place in variety of ways when using ICT (Gardner, 1993). In support of Vygotsky's theory, Gardner suggests that the development of different types of intelligence is possible through ICT. The ICT Test Bed Evaluation conducted in 2006 provides again useful evidence that demonstrate that many teachers opted for ICT to support innovative pedagogy to enhance on the interactivity of learners (Becta, 2006).

The use of ICT tools significantly broadens the learning opportunities for most learners (Ekanayake & Wishart, 2015). Using ICT improves the quality of education (Al-Ansari, 2006), and can accelerate, innovate, and enrich students' learning (Davis & Tearle, 1999; Lemke & Coughlin, 1998, cited by Yusuf, 2005). The visual and interactive tools in computers can make learning exciting for users (Phillips & Scrimshaw, 1997), as they facilitate rapid feedback during discussions and offer sets of challenges to enhance learning (Mann & Tall, 1992).

2.5 Barriers to integration of ICT in pedagogy

The study by TIMSS (2007), reports about the lack of the use of computers in mathematics lessons in countries where the use of computers is relatively high (Mullis et al., 2009). A further report done in the UK by ImpaCT2 in 2003 on students in Key Stage 3 maths lessons stated that 67% of Key Stage 3 students hardly used ICT. Many teachers continue to resist the use of ICT to promote mathematical pedagogy in lessons. While this assertion is true in most cases, the use of ICT in lessons remains problematic. Furthermore, using ICT tools does not necessarily increase attainment (Ellis & Loveless, 2001).

Attainment is not obvious in all National Curriculum subjects when ICT was integrated, especially in Mathematics, Science, and English (Becta, 2004). However, the claim of attainment is seen to be the highest in primary schools rather than secondary schools is absurd (Becta, 2004). Therefore, the claim of the impact on attainment through ICT is 'inconsistent and restricted by the amount of ICT use' (Becta, 2004:5). Kennewell (2000) echoes the view of Joy and Garcia (2000), who argue that learning gains are not achieved solely as a result of ICT integration, but through the amalgamation of ICT with specific pedagogical practices.

The following were among the identified barriers to the use of ICT in mathematics education.

2.5.1 Lack of school and curriculum resources

Kuchler (1998) conducted a study to determine the challenges faced by teachers who employ technological instructional resources in mathematics. The study established that some of these obstacles included the lack of adequate time to set and prepare for a lesson as well as the lack of differentiated resources. Added to this problem is the lack of adequate resources to integrate ICT, as a considerable amount of teachers' time is often spent on finding appropriate resources (Cox, 1993; Johnston-Wilder, 2005).

Despite the benefits of integrated ICT pedagogy, there are many who oppose the introduction of the idea in the school curriculum. In UK, it is widely known that the introduction of the national curriculum in 1988 did not encourage the integration of ICT, as the subjects in the national curriculum remain very traditional (Abbott, 2001).

The initial lack of interest by Her Majesty's Inspectors (HMI) at the early stages of ICT integration into school curriculums reduced many schools' motivation to integrate ICT pedagogy (Abbott, 2001). Graham (1993) agreeing with Somekh (2007) states that the restrictions of the curriculum resources greatly limit the potential for ICT tools to radically transform the learning that takes place in schools.

Abbott (2001) similarly laments that the current school curriculum of some subjects makes it difficult to embed ICT. In addition to the difficulty of integrating ICT into the curriculum, there is the debate as to whether ICT should be taught as a standalone subject rather than through integration (Graham, 1993). Abbott (2001) argues against the approach of teaching ICT as a separate subject and advocates for integration. Despite the importance of 'purposeful assessment of attainment', an integrated ICT curriculum does not easily lend itself to assessment (Kennewell et al., 2000: p.3). Major public examinations are still tested traditionally and do not test pupils' skills acquired through ICT (Kennewell et al., 2000). According to Gillespie (2006), computer-based assessment resources take considerable time, expertise and money to prepare hence there is the reluctance on the part of teachers to promote the practice.

2.5.2 Software and Hardware problems

Kreijns et al., (2013) argue that the lack of hardware support and poor access to technical support present barriers to the successful integration of ICT tools. According to Buabeng-Andoh (2012), eighty percent of teachers' resistance to change and negative attitudes to ICT is due to poor software use. Worldwide surveys in secondary schools also show that the majority of teachers perceive ICT software as the greatest obstacle to ICT integration (Leask & Pachler, 2013). The absence of technical support for software to integrate computer tools successfully is often due to schools' financial constraints, and this further discourages teachers from using ICT (Cox, 1993). Gillespie (2006) argues that where software exists, the over-dependence of the applications has led to slower and unreliable access to internet connections. Often schools' software applications are outdated with no proper maintenance programme in place. The substantial cost of ensuring that children are safe with software online also puts greater pressure on a school's financial budget, as it requires expensive software with continuous technological support in cyberspace (Gillespie, 2006). Most teachers are currently struggling with new software applications due to the high pace at which modern technologies are changing (Barak, 2007; Koehler & Mishra, 2009). Koehler and Mishra (2009) and Barak (2007) argue that the inability of teachers to learn and familiarise themselves with newer digital technologies and computer hardware that keep changing makes it difficult for teachers to keep up with the expected ICT competencies.

2.5.3 Lack of experience with ICT tools

School management teams face considerable difficulties in embarking on school policies for ICT-integrated lessons (Koh & Chai, 2014). A survey conducted by Becta in 2004 revealed that integrating ICT tools into the pedagogy of secondary teachers was becoming difficult due to lack of experience. Mama and Hennessy (2013) also confirm the assertion that the incompetence of teachers is directly linked to teachers' lack of experience. According to Sorensen et al., (2007) the lack of confidence for teachers to use ICT is due to lack of experience. Also, impacting on the experience of teacher's is the limited knowledge of the application of the software, which acts as a hindrance in the use of ICT in the classroom (Koehler & Mishra, 2009). Where hardware devices are used to enhance learning, the inefficient knowledge on how to utilise the software by teachers can impede its integration Clark-Wilson et al., 2014). The inability of teachers to gain the needed experience to deliver an ICT based lessons are associated with a teacher's general interest in ICT. Although many factors affects a teachers experience, among them are the amount of time a teacher allocates to the learning of new ICT tools.

2.5.4 Negative attitudes

The Stevenson Report (1997) in UK identified that due to the resistance towards ICT, its use in schools was 'described as patchy' (Leask, 2001:p.62). Teachers' resistance to educational technologies has been noted for many years, and is evident in their attitudes (Woodrow, 1990). The poor attitudes and approaches to using ICT to facilitate pedagogy are due to lack of 'preparedness to exploit new technologies' (Ellis & Loveless, 2001:p.210). This means that teacher's poor attitude towards ICT-related pedagogies and practices can influence approaches towards ICT usage (Mumtaz, 2000; Watson, 1993). Arguably, changing such attitudes towards practice entails considerable difficulties, largely due to the lack of tailored support, and requires understanding of the experiences of teachers (Ellis & Loveless, 2001).

Negative attitude is one of the known factors affecting ICT integration in schools. Here, 'teachers' Scepticism, lack of confidence and competence with ICT remain centrally important in the pedagogical adoption of ICT' (Somekh, 2008:p.209). Although most studies refute that the attitude of teachers determines how well ICT is deployed in lessons, it continues to be a contentious debate since the use of ICT is 'valuable and well-functioning instructional tool' in education (Becker, 2000, p.29). Individual opinions concerning the use of ICT are often less assumed and remain a difficulty to resolve (Becker, 2000). The changes of attitude required for ICT integration are difficult to instigate, and knowing how to facilitate and support these changes are major concern to institutions (Garet et al., 2001).

Despite the advantages of using ICT in schools to raise attainment, little evidence is known about the relationship that exists between teachers' attitude and the full integration of ICT into schools' curricula. Schiller (2002) affirms that teachers have an important role to play in promoting and supporting the integration of ICT. By understanding the factors that affect the attitudes of teachers, it is possible to bring about the educational changes that are needed in the modern age (Mingaine, 2013). According to Mingaine (2013), teachers' attitude of ICT is important in supporting staff efficiency in relation to integrated learning in schools. Furthermore, He indicated that the understanding of educator's views will increase the understanding of education reforms that supports teachers in their pedagogy.

2.5.5 Lack of training

It was established that teachers lack the professional expertise required for the employment of ICT tools and resources. This is coupled with the shortage of ICT-trained specialists in schools (Cox, 1993). Many critics blame the teacher training for not equipping teachers with the right ICT skills to teach (Cox et al., 1988). In addition, Fu (2013) and Mumtaz (2000) found that lack of training time hinders the use of ICT in schools. The absence of the time to experiment with new skills through ICT initiatives makes it difficult to incorporate ICT applications in classrooms (Fu, 2013; Johnston-Wilder, 2005). According to Gillespie (2006), the prioritisation of schools' initiatives on improving pedagogy over staff professional development has led to less time for experimenting with new ideas. Sorensen et al. (2007) agree with both Koehler and Mishra (2009) and Barak (2007) in indicating that the absence of skilful knowledge in using ICT in lessons discourages teachers to use the software. Generally, the inability of teachers to acquire the needed skills through training or professional development to embed ICT into lessons is blamed on the absence of school clear policies and leadership (Koehler and Mishra, 2009).

2.6 ICT facilitators in education

The reviews reported on many initiatives that teachers perceived to be the main ICT integration facilitators. These supports aim at either removing the obstacles or to encourage teachers to use numerous ICT tools to promote their classroom pedagogy. The section discusses the following support; the availability of ICT tools, experience and knowledge of using ICT, school leadership and the opportunities for teacher training programmes.

2.6.1 The availability of ICT tools

Among the many facilitators of ICT is access to both hardware and software tools. According to Norris et al., (2003), suitable access to ICT tools is a key factor in the effective learning process. For example, interactive white boards facilitate 'visual education in schools' (Ellis & Loveless, 2001:p.210). The availability of digital resources such as digital libraries that makes it possible for teachers and students to access needed pedagogical materials facilities the use of ICT (Cholin, 2005). This availability of such tools when present in the school enhances on teachers confident of sharing good practice. However, many reviews argue that heavy reliance on ICT tools to facilitate learning could lead to the 'loss of hard-won fluent skills' in education (Ellis and Loveless, 2001:175). In some schools where there are shortages of computer suits or long teacher waiting queues on the use of ICT devices, teachers are reluctant to plan and teach using ICT. In situations where internet connections are required for a lesson, often the broad band connections to online resources take ages to download. Such situations often leave both teachers and students frustrated by the long wait for the network connection, hence very easy for teachers to abandon such lessons.

2.6.2 Experience and knowledge of using ICT

The experience of teachers on the use of ICT greatly affects their use in the classroom. The experience of a teacher in the use of ICT acts as a catalyst for change. Experiences acquired of the classroom are by its nature an ingredient to make teachers engrossed in the use of ICT. According to Plomp et al., (2007) teachers who have had the experience of using ICT are by themselves innovators; hence such prior experiences can motivate the use of ICT in practice. An experience user of ICT tools facilities a teacher's creativity through exploration in practice. Notably, ICT is unlikely to be used in practice if teachers have less experience in the required skills. To support the use of ICT pedagogy opportunities should be offered to teachers to either gain deeper knowledge or shaper practical experiences. The case studies conducted by Harris (2002) on three secondary and three primary schools focussing on the benefit of inventive educational practices in ICT, indicated that experienced teachers explores new avenues towards classroom pedagogy. Teachers will feel hesitant to advocate for an ICT integrated lesson where poor experiences have occurred in the past. The use of support staff to assist personnel's to overcome poor classroom experiences can motivate teachers to use ICT.

2.6.3 School leadership

Research by Hammersley-Fletcher and Brundrett (2005) indicates that professional development plays a role in enhancing institutional changes in teachers' pedagogy. A key subject leadership role is to provide 'investment in ongoing professional development, for teachers' (Fullan, 2006:p.4). Glatter (2006) blames institutions for neglecting the challenging needs of teachers by refusing to acknowledge their training needs to be provided by the school leadership. Even though Barker et al. (2010) believe that the desired training of staff enhances their responsibility, the absence of the needed resources to provide the training remains a concern. School leaders must provide the ethos and culture in which ICT integration can thrive. The refusal to identify the prevailing teachers ICT needs, according to Fullan (2006) and Tondeur et al., (2008) is the inability of school leaders to promote a shift in institutional culture. It is also worth noting that when school leaders provide teachers with right and appropriate training, the positive attitudes towards the use of ICT are encouraged (Dupagne & Krendl, 1992).

However, there is a dilemma regarding whose responsibility it is to provide the needed training to facilitate changes, with the failure of institutional training blamed on schools' Human Resources (HMR). Facilitating ICT changes requires creating 'conditions in which all members of the organisation can give their best' (Hallet, 2014:p.17). According to Morgeson et. al., (2010) and Merkley et al., (1997), facilitation is achieved through coaching and mentoring. Using the self-efficacy model, leaders facilitate changes by promoting confidence and self-esteem (Martin-Alcazar et al., 2005; Truss et al., 1997). The ability of leaders to promote opportunities for teachers to 'share good practice' builds their confidence in using ICT (Gillespie, 2006:p.73).

2.6.4 Opportunities for training

At the start of the 21st century, a UK government initiative emphasised the important role of ICT in children's education. This initiative was based on the principle that ICT was vital in education, and insisted that all teachers be trained to use the pedagogy (Stevenson, 1997). A separate survey by DfEE (1998) indicated a strong commitment by the government to support the use of ICT-integrated lessons: 'The government is fully committed to ensuring that all schools and teachers are in a position to deploy new information and communication technology (ICT) to raise educational standards' (DfEE, 1998:p.3).

According to Mann and Tall (1992), providing training can facilitate the use of ICT tools. While having an ICT specialist in the school to assist training is vital, training can also be done through teams, as they provide the 'right opportunity' to facilitate teachers to use ICT (Woodcock 1985:p.87). Working in teams has the benefit of facilitating ICT integration because 'teams outperform individuals acting alone' (Katzenbach & Smith cited in Bush & Middlewood, 2005:p.107). However, developing teams to facilitate any initiatives requires 'time as the team develops and the environment evolves' (Morgeson et. al., 2010:p.12).

Teachers' pedagogical perceptions are tied to their professional attitude, and their perceptions change over time through continuous professional development (Wood & Bennett, 2000).

Although the training done by internal specialists is deemed to be more effective as they are familiar with the school's curriculum and can offer role models for teachers, there is ample research to show that their services do not meet the real needs of teachers (Crook, 1994; Sandoltz et al., 1997).

2.7 ICT and mathematics education

Paul Drijvers (2013), in his work *Digital Technology in Mathematics Education: Why it works (or doesn't)*, noted that digital technology integration confronts the teachers, researchers and educators with a diversified range of questions. What is the ICT potential in teaching and learning mathematics? Which are the conclusive factors in enabling its application in mathematics education? According to Artigue (2002) the failure of teachers to use the required mathematical software applications can limit students' understanding of mathematics.

2.7.1 Constructive reasoning

The importance of incorporating digital technologies to bring about the needed pedagogical and curriculum changes is emphasised in the research carried by McFarlane and Sakellariou (2002) and Rogers (2004). According to the research, students' constructive reasoning has the ability to beneficially develop when technology is incorporated into the learning. Using ICT increases students' constructive reasoning, leading to higher achievement in mathematics (Hanna & Jahnke, 1996). Again, the use of ICT offers the pupils the ability to model and repeat the key process many times to facilitate understanding of mathematical processes. The manipulations performed through ICT are reliable and generally produce accurate results when used for computations (Ruthven & Hennessy, 2003). Oldknow and Taylor (2003), in support of Hanna and Jahnke (1996), argue that pupils' mathematical reasoning is enhanced when different representations of key concepts are seen for the same concept multiple times.

2.7.2 Conjectures and efficient learning

Using digital technologies improves students' performance and advances conjectures (Murray, 1996). According to Murray (2016), to promote conjecture learning, there is the necessity to improve the teaching of mathematics through alternative pedagogical approaches that integrate technology. Since the process of conjecturing to gain understanding depends on time and attention, using technology for learning increases the attention that learners devote to the learning process (Murray, 1996). As Willougby and Wood (2008) rightly point out, learners learn without actually realising the amount of attention devoted to understanding key concepts when computer software is involved.

The use of digital tools increases the efficiency of pupils' learning and improves the value of work that they produce (Ruthven & Hennessy, 2003), allowing the more routine classwork to be accomplished reliably and in a timely manner. The emphasis on speed, ease, and time saving enables much work to be done within a short space of time (Ruthven & Hennessy, 2003), and this improves students' learning involvement in mathematics (Jones, 2011). It is argued that using digital technologies improves speed and accuracy when accomplishing learning gaols, just as mentioned by (Ruthven & Hennessy, 2003; Sorensen et al., 2007; Pierce & Stacey, 2010). This is in contrast to the use of pen-and-paper learning that often leads to many errors and can be time consuming when calculations are involved. The use of digital technologies in geometry provides learners with the ability to interact with geometric objects through 'dragging', which allows them to investigate for example, the properties of quadrilaterals to gain understanding within a shorter time than using pen and paper (Ruthven & Hennessy, 2003)

2.7.3 Guaranteed feedback and interactive support

The use of ICT supports feedback on various processes, testing, and improvement (Ruthven & Hennessy, 2003) An investigation by Trifonas (2008) demonstrated that it is possible to raise attainment in mathematics when students are active with their learning using technological innovations, due to the guaranteed feedback on completed work. The ability to use ICT ensures that instructions on the computer are executed instantly empowering learners to appreciate the differences in learning objects and outcomes. Through the tool, effective comparisons can be made on the same platform to see the differences in the learning for instant feedback (Oldknow & Taylor, 2003; Ruthven & Hennessy, 2003). The ability of the software to produce ongoing feedback motivates children to remain focused on the task. The fast and reliable non-judgemental, impartial feedback provided by ICT encourages learners to form their own conjectures and ideas about the learning (Oldknow & Taylor, 2003). This is crucial for the cognitive development of learners (Baki et al., 2011).

Remarkably, software tools have the ability to provide interactive support for problem solving in mathematics through 'trial and improvement', enabling children to conjecture. The facility to interact with the software provides the capacity for students to investigate and extends thinking time (Oldknow & Taylor, 2003; Ruthven & Hennessy, 2003). According to Ruthven and Hennessy (2003), learners will go the extra mile with their learning when technology is incorporated through interactivity to gain understanding of key concepts. This is possible because by going through more than one procedure, students discover the repeated patterns, and more demanding task is solved quickly (Ruthven & Hennessy, 2003). This use of technology agrees with a move to focus learners' mathematical thinking achieved through instant feedback and promote activities away from just measuring and drawing towards designs investigation and conjecture testing (Sherman, 2014).

2.8 GeoGebra as a digital tool in mathematics education

According to the Key Curriculum Press (2003), the most widely used ICT tool in mathematics lessons is the Geometer's Sketchpad. Even though the Geometer's Sketchpad was the preferred software in mathematics amongst educators (Becker et al., 1999), the use of GeoGebra has the ability to develop inventive activities and geometrical thinking, which makes it an excellent choice by most practitioners (Venkataraman, 2012). At a time when the technology is constantly evolving, it is critical for teachers to incorporate the different dynamics in content delivery, most especially the graphing software.

Currently, GeoGebra (derived from Algebra and Geometry) is one among the highest innovations, open-code ICT tool that is freely downloadable and functions on a diverse spectrum in the system platforms (Ruthven et al., 2008; Jones el al., 2009).

GeoGebra has in the last decade gained popularity among researchers and teachers globally, since it is easy to use and effective in the dynamic mathematics software, which integrates numerous aspects of various mathematical packages. Additionally, it has enhanced the development of an extensive user community owing to its open-source nature.

2.8.1 Merits of using GeoGebra

The open-code maths software offers algebra, geometry and characteristics of calculus in connected environments. In addition, the software offers extensive dynamic geometry concepts in mathematical analysis. The use of GeoGebra has the ability to bring meaning to abstract concepts, through visualisation. GeoGebra aids students in grasping abstract problem-oriented and experimental learning of mathematics. The students can consequently utilise a system of computer algebra in interactive geometric systems, which helps in maximising the cognitive abilities in the best possible ways.

When compared to graph calculators such as the MathPapa, the GeoGebra software is more user-friendly and more efficient in offering compatible interface, commands and multilingual menus. This feature in learning mathematics through GeoGebra reinforces the principle of being learner-centred as well as experiential learning (Kolb, 2015). Additionally, the use of GeoGebra motivates the students' mathematical concepts through guided learning discoveries and a wide scope of presentations (Bester & Brand, 2013). According to Willoughby and Wood (2008), the use of GeoGebra offers learners the capability to go the extra mile of learning without being aware of the time actually spent on acquiring the skills.

The use of sliders and dragging in GeoGebra allows learners to use visualisation to understand key concepts through multiple views (Venkataraman, 2012). For example, students can generate adjustments via manipulation of free objects in different views. The use of GeoGebra provides input of algebra enables the users in generating new objects or modifying already existing ones. Teachers use GeoGebra to stimulate learner's creativity through investigations (Brennan & Resnick, 2012).

2.8.2 Demerits of GeoGebra

Students lacking previous experience in programming will find it difficult to use the input bit with the algebraic commands. Though the simplified commands are not hard to understand, the students will find it confusing. Secondly, the existing research on the use of GeoGebra reports limited research on the impacts on the true achievements in mathematics. Cheung and Slavin (2013) cites that the use of ICT tools including GeoGebra has minimum significance on learner's achievement. This perhaps suggests that to impact learner's achievements other methods of teaching should be considered.

Also although when GeoGebra is used, the application of multiple views enhances learners understanding in mathematics through visualisation, there are limited research on whether the use of the software alone has the ability to accelerate learner's creative reasoning associated with algebra, arithmetic, statistics and analysis (Olsson, 2019). In a realistic perspective, the software possesses in-built support in animations but this is limited in scope. Thus, inclusivity of the modules for animating in GeoGebra has turned out to be a critical technical aspect in any futuristic versions.

2.9 Technological Pedagogical and Content knowledge (TPACK)

The TPACK frame work is used in many studies by teachers to plan and analyse the effective use of ICT in lessons. The use of TPACK allows the combination of factors that a teacher considers enhancing in students' knowledge in learning when using ICT (Schmidt et al., 2009; Mishra & Koehler, 2006). Koehler and Mishra (2009) classify knowledge in three fundamental forms when ICT is used in lessons: Technological Knowledge (TK), Pedagogical Knowledge (PK), and Content Knowledge (CK). Since teachers are very often faced with lots of complexities in technology integration (Monaghan 2004), it is critical to adapt the framework to plan efficiently. The use of TPACK framework aids to conceptualise the teacher knowledge required in integrating ICT in the classrooms (Brennan & Resnick, 2012).

Below is an illustration of the TPACK framework:

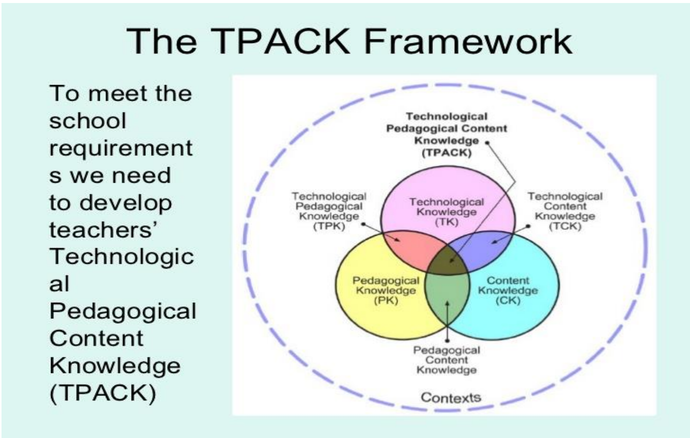


Figure 1: TPACK framework

The use of TPACK aids the teacher to plan for both the integrated and discrete knowledge in the selection of ICT curricular resources. To be more specific, the process of choosing and the using of resources requires a hint of the relevant sources; skills in effectively using the sources; and an idea of the features of the relevant resources. Additionally, knowledge of the specific hardware and software is required in enabling the teachers to comprehend the options from which they could select and efficiently use in lessons (Clark-Wilson et al., 2014).

In the circle in figure 1, the areas that overlap are three kinds of knowledge combination, which can be explained as follows:

- Pedagogical content knowledge (PCK): this defines the knowledge that teachers possess concerning their content (Shulman,1986)
- Technological pedagogical knowledge (TPK): this is a skill set that enable teachers to identify the best technology in supporting a specific educational approach (Mishra & Kohler, 2006).
- Technological content knowledge (TCK): this denotes the skills that were acknowledged in which the teachers recognise the finest set of skills in using technology to support students to learn the subject content (Mishra & Kohler, 2006).

Our main area of interest will be on the content knowledge, since most of our discussion on integration of the dynamic graphing software will be firmly established from content knowledge.

2.9.1 Content knowledge (C)

Content knowledge is associated with the definite matter of interest that is to be learned or taught (Rowland et al., 2009).The question that comes to mind is how the various kinds of 'content' knowledge are applied by mathematics teachers alongside incorporation of the dynamic software in the teaching of a mathematical topic?

2.9.2 Pedagogical content knowledge (PCK)

Intuitively, it is with no argument to believe that a good teacher should have detailed subject knowledge. Many policy makers and research over time has described accurately what knowledge a teacher should have to be successful (Ball, 1991)? To answer this same question, Ball (1991) argue that inadequate insight in the understanding of the subject matter adds to poor numeracy skills and can contribute to a lack of curiosity in teaching maths. The perception of learning mathematics is further reinforced through the heavy dependence on textbooks listing 'hints' rather than conceptual development. No wonder many studies on this subject still consider what constitutes a reputable teacher to be implicit and reflect teaching as a vocation based on the application of common sense with less involvement of professional education.

Schulman (1986) theorised pedagogical content knowledge (PCK) as an exclusive realm of teacher knowledge including numerous facets of mathematical knowledge relevant for teaching. Regarding PCK, he proposed to justify the principal role of subject content knowledge in the quality of teaching together with pedagogical knowledge. In Shulman's (1986) perception, PCK is associated with both having the knowledge of the subject content and the ability to support others to understand it. Krauss et al., (2008), by refining the views on Shulman perception of PCK, identified three key scopes of PCK: the teacher's mathematical knowledge; the interpretation of students' knowledge and thinking; and knowledge of numerous intelligences on mathematical problems. Ball (1991), in their definition, also takes account of the subject knowledge, teachers view comprising the structure of the curriculum and the gradual introduction of concepts.

Eisenhart et al., (1993) did not find Shulman's PCK concept supportive and dismisses the difference between pedagogical content knowledge and subject. Eisenhart et al., (1993) uses conceptual knowledge and procedure as constituents of teacher knowledge. Cochran et al., (1993) use pedagogical content knowing (PCK) to emphasise on PCK, an inherent knowledge associated with a specified teaching context. Shulman's thought was difficult to justify with empirical data and is again seen as a theoretical concept. To overcome the difficulties with Shulman's constructs, Rowland et al., (2005) introduced the theory to advance an empirically founded taxonomy, the 'Knowledge Quartet', which differentiates mathematical contents into foundation, transformation, connection and contingency.

2.9.3 Limitation of TPACK

The first supposition of the use of TPACK is that technology, pedagogical and content knowledge happen in segregation and can be assumed through separate overlapping circles as a symbol of empirical realism. This further assumes that technological, content and pedagogical knowledge happen in a distinct process. However, the content expressed in the form of learning is impossible in the absence of pedagogy and the ability to present explanations of key concepts is naturally represented in pedagogy (Archambault & Barnett, 2010). This makes Shulman's framework seem unreasonably complicated when the common association that exists among content knowledge structure and the equivalent pedagogical knowledge are accommodated in a single frame (Archambault & Barnett, 2010).

Added to this is the second assumption of TPACK on the content knowledge (CK). According to Shulman, the authority of content knowledge depicts one of the key constituents of digital education and must be viewed as one of the key concerns of education. This held view is not justified and expounded with convincing reasons. According to Segall (2004), the act of recent education is rather alarmed on the attainment of capabilities, individual resources and development skills, and not content as such, although content is defined by key concepts and cannot easily be disconnected from informal epistemological admittance.

Another limitation of TPACK was the assumption of an ideal absent of actors and goals in mind when the framework is in practice. This final assumption perceived that it is confusing by distinctively locating boundaries around T, P and CK as clear regions when planning teaching and learning activities with a digital media learning environment (McEwan & Bull, 1991; Archambault & Barnett, 2010). The assumption that key elements of learning procedures are also separated from, for example, key educators including teachers makes TPACK a less student-centred model, which is a serious concern for educators to ponder on when deciding whether to use it.

2.10 Chapter Summary

This section discussed the reasons for education in schools in relation to the mathematics education curriculum in UK. To understand the impact of ICT in education, we discussed the role of ICT in education in particular in mathematics education. The chapter was concluded with the review on the use of ICT tool GeoGebra, as digital tool to teach mathematics together with an identified Framework for ICT lessons.

CHAPTER 3 RESEARCH METHODOLOGY

3.1 Introduction

This chapter identifies the research method employed for this study including the type of data that was gathered and how it was analysed to generate the results for the study. This section begins with the justification of the rationale and purpose of the chosen methodology. The research approach is further discussed together with the chosen design. Also mentioned here is the examination of the data together with the selected ethics for the research. The research employed mixed methodology approach; quantitative and qualitative research designs.

The following research questions guide this study:

- What are the benefits of ICT in Mathematics education?
- What are the barriers to ICT in mathematics education?
- What factors will facilitate ICT in Mathematics education?

3.2. Research design

The descriptive enquiry design dominates this study since it defines the characteristics of the respondents. The method is suitable because it allows the researcher the benefit of conducting a detailed analysis as well as appreciates the philosophies of the participants using their own experiences and words (Bernard & Bernard, 2012). This method allows the researcher to make deductions dependent on their own opinions of the practice (Rubin & Babbie, 2012). In sharp contrast with the design that is exploratory, the researcher discovers new ideas in the study and connects the origin and the outcome of these issues (Bernard and Bernard, 2012). When descriptive design is used, the researcher is able to further study the research question before establishing any conclusions (Kreijns et al., 2013).

Two aspects guided the research design employed and these are the use of TPACK and Knowledge Quartet. These two frameworks have been explained and incorporated into the research design as follows;

3.3 Lesson Design: TPACK and KQ

The TPACK framework guided the design in aiding to express the knowledge content as well as the application of the technology. The PCK (pedagogical content knowledge) framework offers the teacher the opportunity to use the subject knowledge in designing the lesson. This was achieved by consulting the schemes of work to understand the key concepts to be presented in the lesson. Furthermore, there was the need to conduct an initial audit on the use of the technology within the subject knowledge before introducing new concepts. Finally, a consideration was given to how to explain the subject content to empower learners to acquire the needed mathematical knowledge.

Another aspect of TPACK framework that was deployed was the technological pedagogy knowledge (TPK). The knowledge of TPK reminded me to think clearly on own actions as a teacher and communicator of information to learners. Teachers in this position are at the forefront of correcting errors in teaching and education practice (Hadjidemetriou & Williams, 2002). To support both individual and whole class discussion learning, I embedded in my lessons questions to promote learners' thinking skills, vital to extend learners thought processes (Malibar & Pountney, 2002; Ruthven & Hennessy, 2003).

In planning for PK and PCK, students were equipped to carefully use a correct shared dialogue. Throughout the planning and the delivery of the lesson, opportunities were offered to students to discuss their work with other students and receive feedback on their completed work. Recognising the diverse needs of each learner enabled me to physically approach students to obtain feedback on the completed task. Much learning takes place when immediate assistance is at hand whilst completing a mathematical task (Saudelli & Ciampa, 2016). The designed worksheet was differentiated by task to ensure that each student benefited from the lesson. Avoiding generalisations and accommodating for individual learners' needs encourages learners to thrive in group studies (Mavrikis et al., 2012). Specific questions such as 'Why' and 'what', open-ended questions, were utilised to prompt class discussions and debate about the completed work.

3.4 Design tool

I used GeoGebra as a device to improve the delivery of my pedagogy; I acquainted myself with the expectation of the functions to be able to support learning. Also, I ensured that the software was running on all the computers before the lesson. This knowledge and awareness of ensuring that my technological understanding of how to use GeoGebra to teach was expressed in all the lessons, a vital knowledge to ensure an effective technological integrated learning (Stupel & Ben-Chaim, 2017). Saudelli and Ciampa (2016) state that many unexpected challenges in using technology can be avoided when, during lesson planning, teachers anticipate and plan for them.

The knowledge of learners' background for John (2006) was vital for an effective lesson planning. The knowledge of students' backgrounds for my lesson planning included knowledge of their experiences and history of ICT used. I further consulted the school data on all the learners to be able to plan the different preferred learning styles in the class. In particular, the knowledge of students' previous digital use including the use of GeoGebra was considered for my planning (Godwin & Sutherland, 2004). The background information of most of the students indicated little use of ICT so the starter activities were focused on support tasks to ensure a scaffolding approach into learning. Added to this knowledge was the need to promote connectivity and collaborative learning. According to Gerlach (1994) learning takes place naturally when learners are offered the opportunity to discuss in groups.

3.5 Interpretivism

The research is structured within an interpretative paradigm. The context of study is within an educational setting. The interpretive paradigm is the most appropriate because the study aims to understand the perception of the participants (Willis, 1995). This approach stresses on knowledge socially acquired from participants (Sexton, 2003).

The reason why the method has been considered and used in this research is, firstly, because all the analysis and interpretations made in this study are primarily based on the researcher's personal experience and the perception of teachers through questionnaires with integrating ICT in lessons. Secondly, interviews were conducted to support data from the case studies and questionnaires. Also data was gathered from the literature to support the extracted primarily data.

The interpretive paradigm used in this research is reinforced by the interpretation attached to the collection of information on own practice to interpret it (Sexton, 2003). The interpretation process consists of making meaning of the information obtained by drawing inferences and by establishing inferences between some abstract patterns (Aikenhead, 1996).

3.6 Research approach

First, primary data was collected as a means of understanding own practice with regards to digital integration in teaching circle theorems and linear functions. Thereafter, the data on questionnaires and interviews were gathered. Primary data has the ability to describe the events that really took place in the lesson (Wallace, 1991). The main sources of data collection for researchers on own practice, according to Richards and Lockart (1996), are lesson plans, reports, audio or video recordings, observations, and questionnaires on teachers' own practice. These sources allow a teacher to critically analyse their own practice in reference to subject literature. Using different episodes and case studies in the lesson allow the teacher to reflect on own practice with a view to improving (Richards & Farell, 2005).

According to Kvale (1996), research conducted within the context of the qualitative method uses interviews to understand world realities. The data collected through the interviews will enable me to make generalisations of the entire population of students in my class. Using the semi-structured interviews data and teachers will provide the researcher the information needed for the analysis (Brinkmann et al., 2015). Using interviews with has the added advantage of allowing the researcher the time to probe the participants in depth for the required information. However, the limitation here is that it requires plenty of time to conduct interviews and to analyse the transcribed data and it can be expensive and time consuming when a large amount of data is to be analysed (Halcomb & Davidson, 2006; Seidman, 2013).

3.7 Methodological approach

The research adopts both the explanatory and the descriptive research approaches under the qualitative and quantitative methodology. A mixture of these methods offers the investigator the ability to ascertain the phenomenon under study (Saunders, 2012). Burns and Grove (2003, p.19) describe a qualitative approach as 'a systematic subjective approach used to describe life experiences and situations to give them meaning'. A qualitative approach lends itself to a 'holistic description of events, procedures, and philosophies occurring in a natural setting needed to make accurate situational decisions' (Stainback & Stainback, 1988, p.88). It is because of this reason that the qualitative approach has been employed in this research; the data gathered on case studies and used in this study is descriptive and is from the owner's experience. The subjective view on a participant is based on 'world view' with the aim to present a holistic picture of an individual's perceptions and interpretations on a concept (Yin, 2016).

Although both the mixed approaches were used, the quantitative was the dominant methodology. Choosing the qualitative research over the quantitative research offers the researcher the ability to deploy the perceptions of participants in relation to known theories (Conti, 2006). Also, by depending on multiple sources, the researcher obtains contextual information to make the results more significant and less subject to diverse interpretations (Fraenkel & Wallen, 2008). Furthermore, particularly, that researches accomplished through the quantitative paradigm do not give thought to subjective and social phenomena, compared to qualitative researches, where participants are studied in their usual environments, the mixed approached was considered (Conti, 2006). Qualitative approaches have the added advantage because 'the researcher can study dynamic processes (i.e. documenting sequential patterns and change)' (Burke et al., 2014, p.20). Furthermore, within the qualitative research paradigm, prevailing knowledge or perception has significance within a specified circumstance or context (Hadjidemetriou & Williams, 2002).

Although qualitative research permits in-depth examination of the participants, the validity of the research can be rejected (ibid.). According to Silverman (2006), the problem of reliability arises through definitions and the use of longer narratives. This problem arises perhaps mainly because of the way the 'researcher goes about categorizing the events or activities described … many studies provide the readers with little more than brief, persuasive, data extracts' (Silverman, 2006, pp.44–47). The mixed approached was favoured because their benefits complements each other to produce a more object conclusion even though, the use of quantitative approach often involves heavy calculations which may be expensive and time consuming.

3.8 Data collection

Since no single data source has the ability to provide the researcher with the entire accurate and whole picture of the exact occurrence of events, multiple viewpoints were sought to represent the variety of data collection methods to present the diverse features of the same research problem or question (Waters-Adams, 2006). To respond to the research question about the impact of ICT in mathematics education, data collected entailed lessons showing instructions on how to employ the ICT tool to teach the various concepts of circle theorems and linear functions. A detailed investigation of the literature around the role of digital technology in education was also included in the review. Recorded videos about the use of GeoGebra application in the lesson were observed and analysed.

To aid the analysis, the lessons reflections were examined and scrutinised. The reflections on the lessons allows the researcher to step back and look at what has been achieved and what could have been changed to improve their performance. According to Mizuno et al., (2016), a reflection offers practitioners a view of their lessons for critics and analysis. Various methods are employed for lesson reflection and the one used in this study is the student observations strategy. This involves getting feedback from the students regarding what has been delivered to them. Alternatively, giving the students a practical exercise that they are required to complete can tell the teacher about what they learned and what they did not understand (Mizuno et al., 2016).

During the data collection process, Mizuno et al., (2016) mentioned that understanding the language and the body expression of the research participants helps to understand their perceptions as well as their values regarding a particular matter of investigation. It ultimately helps the researcher to analyse the data accurately.

3.9 Questionnaires

Questionnaires were used to collect data from 40 maths teachers to analyse and present their views. The uses of questionnaires were used to supplement the information obtained from own practice through the case studies. Apart from being an effective way of obtaining data, using questionnaires has the benefit of obtaining a large of data within a limited time constraint. The results of questionnaires when analysed allows more scientific comparisons than results from other researches (Popper, 2004). However, most phenomenologists claim that the use of questionnaires in researches are unrealistic since by using it , a researcher requires for only limited information without any elaboration. Also, it is not obvious to know whether the respondents answered the questions truthfully or not when questionnaires are used in researches (Ackroyd, & Hughes, 1981).

3.10 Participants

3.10.1 Students

The participants were students from my own class. The selection of students was purely due to convenience taking into consideration gender and academic achievements. Ten students in the class were approached for the feedback on the lesson. The views of the sample students were analysed and deductions made representing the entire population. In defining the implication of the perception of the sample on the entire population, Polit et al., (2001) indicated that samples are used in researches because they are seen as ‘a proportion of a population’. A smaller sample is justified because in researches involving qualitative methodology, it is typical ‘to study a few individuals or a few cases’ to make generalisation (Creswell, 2011, p.209). Also, Ritchie et al., (2003) recommend the need for a small sample size within qualitative research because phenomena only need to appear once to be part of the analysis. The advantage with the small sample is that it saves time and is convenient (Creswell, 1998).

3.10.2 Teachers

To ensure that the information from participants satisfied the purpose of the research, ten teachers were selected from different social and economic backgrounds for the interviews. Whilst a total of 50 questionnaires were administered to selected schools, only 40 were received and analysed. The teachers were selected from both comprehensive state and private schools with varied teaching experiences. The information collected from teachers taking into consideration age, sex, socio-economic status, experiences of service mirrors the larger population that the study intend to represent (Center, 2015).

The table below states the teachers’ positions, and other vital information used to aid the analysis (Thorpe & Tran, 2015).

Table 1 Mathematics teachers: interviews

Teachers Code	Length of service (years)		Age (Years old)	Gender	Appendix
	Years in service	In school			
T1	10	5	30-35	Female	15

T2	15	10	30-45	Female	16
T3	8	3	30-35	Female	17
T4	10	4	30-45	Male	18
T5	20	9	30-35	Male	19
T6	15	4	35-45	Female	NOT INCLUDED
T7	8	8	30-45	Male	
T8	12	7	35-40	Female	
T9	14	5	30-35	Male	
T10	10	8	30-45	Male	

Table 2 Mathematics teachers: Questionnaires

Gender	Frequency	Percentage	The data collected reflected mixed gender composition of 60% male teachers and 40% female
Male	24	60	
Female	16	40	
Total	40	100	
Age of teachers			
Under 30 years	6	15	The results demonstrated that 85% of the teachers were above 30 years
30-40	24	60	
40 and above	20	25	
Total	40	100	
Teaching Experience			
Under 5 years	6	15	The results indicated that 60% of the teachers had between 5 to 10 years of teaching experience.
5 – 10 years	24	60	
10 and above	10	25	
Total	40	100	
Qualification			
Diploma	10	25	The research revealed that

Degree	24	60	75% of the teachers had a bachelor's degree and above, very qualified to teach the subject.
Masters	6	15	
Total	40	100	
ICT Experience			
Below average	12	30	The results demonstrate that 30% of the interviewed teachers had below average ICT use.
Average	24	60	
Above Average	4	10	
Total	40	100	
ICT in teaching Mathematics	Frequency	Percentage	
Never	2	5	The results indicated that 95% of teachers use ICT in maths education.
Sometimes	32	80	
Always	6	15	
Total	40	100	

3.11 Materials/research tools

The materials used are found in Appendix 5. Three types of interviews were considered before deciding which method would best suit the study on understanding the impact of students' learning with an integrated ICT tool. The three distinct types of interviews considered were the structured, semi-structured and the unstructured. The semi-structured interview was preferred over the other types of interview due to its ability to represent a large amount of information (Kvale, 1996). Structured interviews are used where the research is exploratory and it is difficult to take account of a catalogue of potential pre-codes research (Mauthers et al., 2002).

3.12 Recording interview data

All data obtained throughout the interview were carefully recorded on audiotape together with documented field notes to enrich the taped discussions and to aid analysis. Note-taking is important to capture the interview and it acts to supplement the data stored on the audiotape (Holloway & Wheeler, 2002).

The subsequent issues were reflected when audio-recording to safeguard a fruitful interview: I obtained consent to use the tape recorder at the beginning of the interview. Again, all the participants agreed to its use at the start of the interview. By using the tape recorder, the researcher could maintain eye contact to preserve the applicants' words throughout the data collection (Holloway & Wheeler, 2002). The tape recorder and the electric power point in the room were verified before the interview to guarantee a good working condition. An additional back-up with batteries was available in the unlikely event of an electric power cut. Finally, all audio tapes used were properly addressed with dates, name, gender and the number of the participants interviewed to ensure that the right interview was analysed at each stage of the data analysis.

3.13 Recording lesson episodes' data

Recording one's own practice experience ensures that the research is as objective as possible. Although studies on own practice examine life in its usual setting, caution must be maintained not to extinguish the interactional honesty of the setting (Bateson, 1979).

To be able to keep track of the information needed for the research, it was vital that all data are collected as one goes along (White, 1998). Philip White, commenting on the importance of recording every detail when researching yourself, reiterated that collecting all possible data on own practice has the added benefit of the ability to examine the information with the available field notes. By immediately recording the data as it happens, this grants the researcher the creative, complex, and profound understanding of practice to maintain a deeper or more qualitative level of information than when collected at a later date on previous conventional sources (White, 1998). In analysing the data, it is a good practice to always keep in mind, as one proceeds with the analysis of data, that such research is a dynamic, and cyclically recursive, process of inquiry (Ladkin, 2004).

3.14 Case studies

The uses of case studies are widely noted in social sciences researches. The design of a good case study aids the researcher to collect, analyse and present data in a fairer manor (Yin, 2009). The use of case studies in both quantitative and qualitative gives the researcher the advantage to understand changes due to complexities related to personal procedures that have appeared through broader communal context (Cronin, 2014). However, the disadvantage here is that, the data obtain from case studies can be complex hence presenting researchers with the difficult to analyse and present their findings especially due to multiple cases inherited in the study (Soy, 1997). Most researchers use case studies to improve and introduce new theory in order approve or refute an existing theory, to explain a situation, to explore or to describe a phenomenon or an object. Case studies are also used for empirical and descriptive research (Solberg et al., 2009). In descriptive researches case studies are used for observed situations whilst in empirical researchers they are used for enact among parties to increase knowledge of the actions. This research combines the benefits on both the descriptive and the empirical approaches.

3.15 Data analysis methods

Rubin and Babbie (2012) cite two major data analysis methods, quantitative and qualitative data analysis. This research uses both the qualitative and quantitative method analysis. In a case of qualitative data analysis, the researcher generally obtains the response with the help of semi-structured interviews and analyses the response along with the secondary data available from other journal articles (Blumberg et al., 2014). Analysing own research involves the ability to be objective with your own findings as much as possible (Hubbard & Power, 1999).

Again, by using the qualitative data analysis technique, the responses of my actions and lesson episodes were matched with the information stated in different scholarly journals and articles (Blumberg et al., 2014). Since the study is focused on analysing human beliefs and perceptions, the choice of quantitative data analysis technique was inappropriate to suit the purpose of the case studies, since the data collected was purely non-numeric (ibid.). Blumberg et al., (2014) state that, generally, quantitative techniques are chosen in case of determining the percentage rates of satisfaction and usage. Using the qualitative analysis techniques allowed the researcher to analyse the opinions and beliefs of the respondents (ibid.); hence, the choice of qualitative analysis technique over quantitative technique for the case studies.

Analysing one's own research data required me to think carefully with the aid of the TPACK framework. This process involved visualisation and scrutinising my own practice without pre-judging own efforts. This procedure enables a researcher to act as detective, observe, contemplate, inspect and look for configurations and patterns alive in their own data (Pine, 2009). Also, the objective of the study is to critically analyse the data in the light of the Knowledge Quartet to provide explanations as close to the available data and experience achieved when the researcher avoids the predetermined dichotomies, held views are pronounced as axioms by specifying rigid formalisms about how to conjecture and research (Pine, 2009).

The analysis used the thematic content analysis approach to analyse the data. The use of thematic content analysis ensures that qualitative data for the case studies are analysed completely using patterns, and broad themes, instead of counting and analysing statistically. The decisions around the selected size of circles as well as the level of unit of overlay were grounded on specific trends recognised in the data. The four intersecting knowledge regions and the three main knowledge units (content, pedagogy, and technology) of the TPACK framework model were used as major coding categories.

3.15.1 Triangulate data

Triangulation consisted of the use of numerous data sources. To maintain validity of results, a researcher combines triangulation with reflexivity (Banford, 1996). The act of reflection consists of critical awareness, conscious by individuals about their specific prejudices, preferences, and expectations both before the study starts and as it develops (ibid.). The research question was studied from different perspectives before arriving at the conclusions. The different data sources identified for the triangulation included lesson episodes, the reflections, lesson plans, interviews with students and samples of students' work. According to Ferrance (2000), studies that involve own practice usually consider an extensive variety of sources to guarantee the researcher the needed validity.

To obtain the right information, questions were continuously asked to ensure that the data collected was indeed suitable to answer the research question (White, 1998). By examining the data collected against any previous data held, this offered the researcher the bases to decide which information should be focused on for the study (Pine, 2009).

3.15.2 Coding and analysis teachers responses

Coding is a systematic way to combine and organise data based on relevant themes, topics, ideas, terms, concepts, keywords, terms, and phrases (Albugami, 2015). Coding is a very useful process that helps to retrieve the most salient data from a large data set, and helps the researcher to compare and analyse the data properly (Albugami, 2015). Codes from the categories enable the researcher to manage data by labelling, storing and retrieving it according to the codes (Miles & Huberman, 1994). As the key elements of the data are identified, it was possible to categorise the data for coding. Designing a methodical method to analyse the data develops through the realisation of the requirements of the research question.

Coding becomes possible through categorisation and clarification of the data to determine how to organise the data results, in an order of significance and frequency (Pine, 2009). By using the existing predefined coding (Appendix 3), colours were used to highlight the frequency of the information associated with the predefined codes of TPACK and the Knowledge Quartet. Using colour coding enhances the researcher's ability to discover similar trends for the analysis.

3.16 Research ethics

Throughout the research, the involvement of human respondents in the project necessitated the need to abide by the ethical guidelines as prescribed by BERA (2011). Prior permission from the respondents was taken from both teachers and students and the intention of the data collection with the use and the aim of the research was fully explained before conducting the research (Bernard & Bernard, 2012). As per the Data Protection Act 1999, further care was taken to maintain the privacy of the personal information collected from the participants, which included name, date of birth, gender and other details. This information was stored on a secured data stick with password protection.

Moreover, care was taken to avoid any kind of coercion or pressure over students so that unbiased views regarding the questionnaire could be obtained (Blumberg et al., 2014). During the data collection process, prior permission from the students and teachers was obtained before any questions were asked (BERA, 2011). Also, the need for the participant to withdraw at any point of the interview if not comfortable with it was explained (Blumberg et al., 2014). Finally, together with the questionnaires, all participants were granted anonymity and confidentiality by the use of ethical codes (BERA, 2011).

3.17 Limitations of the research

While the current study has presented findings that have an impact on the usage of technology dynamics to teach linear functions and mathematics in general, this implication cannot be generalised. The scope of my study poses a limitation on the findings in that it solely depended on two lessons, interviews and questionnaires on 40 participants from 5 schools. As such, it may not be applicable to many other classes and schools that uses technology dynamics. Additionally, the suggested implications might not apply if the same research is conducted but with a different class and a different technology. Also, research on my own practice may not produce the objectivity needed for this study. The following is a list of challenges that were experienced during the pursuance of this study:

- Difficulties of objectively scrutinising own work and separating myself from the research.
- Inadequate documentation and transcript of the semi-structured interview results due to time pressure and school's end of year activities.
- Difficult of obtaining all the questionnaires as expected.

3.18 Chapter summary

In considering the methodology approach for the research design, it was imperative to use both the qualitative and quantitative methodology approaches within the interpretivist paradigm. The data obtained from the questionnaires, the semi-structured interviews and the case studies provided the data for the analysis. All materials used with their ethical consideration were conducive for the chosen research design.

The analysis of the findings emerging from the application of the design questions are discussed in the next section.

CHAPTER 4: DATA ANALYSIS

4.1 Introduction

This chapter of the study addresses the analyses of the results collected from the semi-structured interviews and questionnaires. Both quantitative and qualitative analytical tools were used to analyse the responses from the teachers. All data collected from the participants during the interviews was analysed as soon as the researcher was conversant with the data (Braun and Clarke, 2006). The audio from the semi-structured interviews was transcribed and read many times in order to develop a coding scheme (Bogdan & Biklen, 1992). The initial codes were then used to generate themes to aid the content analysis of the data (Albugami & Ahmed, 2015), and subsequently answer the main research question: 'what is the impact on ICT integrating in mathematics education?'

The following research questions guides this study:

- What are the benefits of ICT in Mathematics education?
- What are the barriers to ICT in mathematics education?
- What factors will facilitate ICT in Mathematics education?

4.2 Rationale of analysis for the interviews

The data analysis was done using illuminative analysis, within the interpretivist paradigm. By adapting the constant comparative method, the data from each interview was compared with the elements, phrases, sentences, and paragraphs of the other interviews (Thomas, 2009). The analysis process consisted of familiarising myself with the data. A general pattern emerged from listening to the tapes, transcribing interviews from tape to paper, and reading over the written transcripts several times (Rubin & Rubin, 2005).

Following the formal process of analysing the qualitative data, the data obtained was coded and main ideas in the data identified. The process continued with gathering identical information together in categories to ensure that similar themes and ideas were connected (Rubin & Rubin, 2005). Organising the similar themes and ideas in a table also provided a useful snap short of the information to be analysed, as 'valid analysis is immensely aided by data displays that are focused enough to permit viewing of a full data set in one location and are systematically arranged to answer the research question at hand' (Miles & Huberman , 1994:p.432).

4.3 Possible outcomes

By analysing the findings from the case studies, interview from teachers in conjunction with the literature review, I expect to find that the use of ICT has a strong impact on mathematics education. It is also expected that the teachers' perceptions of the barriers to ICT integration are related to their experience of using ICT. Finally, I expect to find that teachers' will report that the school curriculum is a barrier to the use of ICT in mathematics education. To improve the impact of ICT integration in mathematics, I expect teachers to recommend that schools must encourage ongoing professional development to boost their ICT confidence. The research is somewhat limited by some teachers' difficulty in providing answers that honestly reflect their perceptions on the subject, and the inevitable bias of participants towards the use of ICT.

4.4 Dissemination

Ethical considerations are considered in the dissemination of the findings (ESRC, 2010). According to BERA (2011) researchers must refrain from publishing unethical materials and participants should be made aware of what is reported. The dissemination tools considered for this study includes peer review papers, research reports, and presentation of report at both national and international reviews. A report of the findings will be presented first to participants and will be encouraged to comment on the study. Furthermore, the report will be sent to interested schools for deliberations and recommendations on the views raised on the perception of ICT pedagogy.

Samples of the records of the semi-structured interview schedules with transcripts are seen in Appendices 15 to 19.

4.5 Case study 1: Circle theorem

The target audiences are students in year 9 (14-15 years old). I will discuss the rational for choosing the TPACK and KQ framework for the plaining, delivery and the findings from the taught lesson. The reflections and the interviews with students on the impact of the use of

ICT form the bases of the analysis. I planned to use both open and closed-ended questions, as advocated by Malibar and Pountney (2002), whilst aiding whole-group discussions through collective learning (Ruthven & Hennessy, 2003; Godwin & Sutherland, 2007).

For the purpose of the analysis, the students in the class were put into groups. Group A consisted of students S1, S2, and S3, who shared a computer. Group B included students S4, S5, S6, who shared a computer as depicted in the table 3 below.

Table 3: Computer sharing

S1	S2	S3	S4	S5	S6
Group A			Group B		

Appendix 1: Demonstrate the application of using Technological, Pedagogical Content Knowledge to design the Lesson with examples of activities that were considered.

Appendix 2: Activity Sequence, including the objectives of the lesson with the task to be completed.

Appendix3: A detailed analysis of the discussion on each circle theorem concept.

: The use of GeoGebra to demonstrate how to resolve the misconception on the topic.

The following were the series of planned task to aid pupils understanding of each key concept.

- Task 1: The angle at the centre of a circle is twice the angle at the circumference of the circle from Task 2: Angles inscribed in a semicircle are right angles the same arc.
- Task 3: Angles in the same segment are equal.
- Task 4: Cyclic Quadrilateral Theorem
- Task 5: Alternate Segment Theorem
- Task 6: Perpendicular & chord
- Task 7: Angle between tangent and radius

4.6 Case study 2: Linear functions

The aim of this study is to establish how ICT is used in teaching linear functions. Using GeoGebra, several episodes can be analysed to determine the effectiveness of the learning.

4.6.1 Lesson Episode 1: Drawing Graphs

The aim was to use Geogebra to draw graphs by exploring how to adjust the y- axis, x - axis and to edit in the graphic view. This was a one-hour introduction lesson in GeoGebra. The activities of the lesson were followed by the completion of a worksheet whereby each student used the software in generating certain example of a graph family in the form: $y = c$, $x = c$, $y = x + c$, $y = mx$, $y = x + 2$, $y = x – 3$, and then record them on the worksheet (Appendix 4 and 6).

In order to answer the research question, what is the impact of teaching mathematics using ICT? Over the major part of the lesson, the format of the activity alternated between short quizzes and the whole classwork discussions. The use of the software enhanced the accessibility of the tasks and made it less tiresome to the students to complete the classwork. Also the use of software function assisted the teaching in the interpretation of the variable in the linear graphs through colour coding. Henceforth the use of different colours of the graphs helped the students to see which colour goes with which function to aid the interpretation of drawn graphs. In the lesson, I posed a speculative question concerning the sloping of the lines in different ways, which then led to similar trailing episodes inserted in conventional investigation alongside introductory review.

4. 6. 2 Lesson Episode 2: Slope and y-intercepts

In this lesson, I introduced how to draw a linear graph by introducing students to how to construct two distinct points followed by joining the two points together to produce a straight line. The relation between the line drawn was established and changed into a function or equation in the form of **y = mx + c.** To transform the line into a linear equation, I guided the learners to create an object to show the equation of the line already drawn to appear on the screen by inputting this equation of **y = x + 1,** in the input box (Appendix 6)

The next step was to interpret the characters in the equation. Using a cursor in the grid, I was able to help the learners to identify the required interpretation on the equation of a line and also identify the constant of the function as the point where the line intercepts with the y axis.

While at this point, I was able to make changes to the y-intercept by moving the line or the points to achieve intercepts such as 0.25 and 2.

In the next step, I explained the significance of the slope in the linear function and how it is determined using the GeoGebra software. By making shifts using the drawn line, it was possible to explain and demonstrate that the slope was determined by the movement to the right from the y grid at the intercept and a movement to the position of the value of the x in the grid. By illustrating that the movement of the steps to the right can be used to describe the ratio in the gradient, it was possible for the learners to appreciate the meaning of gradient as expressed in ratios. After guiding the students to determine different slopes including ½, 3, 5, and so on, I then guided the students to work on their own examples (Appendix 4 and 6).

In order to answer the research question, what is the impact of teaching mathematics using ICT? I physically moved around the class to find out what the students were able to complete in the lesson. Also, key questions were used during the lesson to be able to determine the barriers in their learning. By using questions such as 'What is going to be the next stage?' I was able to keep most students focused on the task. Similarly, the brief pause of lesson review and questioning provided time for students to think through on what the next steps would be before demonstrating the next activities. Students reported that the use of questions facilitated their understanding of the topic better.

4.7 Teachers Interview questions

In order to answer the research questions exhaustively, codes were further grouped into the following categories with their respective interview questions:

4.7.1 Part 1/Code 1: Benefits of ICT pedagogy

Interview question 1: What do you believe to be the benefits of using ICT in mathematics education?

In the answer to the above question, T1 (Appendix 15) and T7 stated that they believe that the integration of ICT into their pedagogy helps them to reinforce learning within the classroom. Both also mentioned that if ICT were included in the pedagogy, then there would be higher student interest and more active participation in learning. T3 indicated that 'interactivity and instant feedback to class work' was possible through the use of ICT (Appendix 17). T4 mentioned that due to the interactivity provided by the integration of ICT, it had become easier to supervise students in the classroom (Appendix 18). The use of ICT has the ability to keep most pupils on task, which has improved pupils' behaviour for learning and enabled better class room management (T4, Appendix 18).

According to T4 and T9, the integration of ICT tools assisted the use of modern technologies. T1 indicated that modern technology is 'incredibly useful and speeds up the research process' when students are looking for new information (Appendix 15). For example, most teachers mentioned that with the help of projectors and interactive white boards, important learning videos were embedded into the pedagogy. T3 and T6 further stated that integration of ICT tools helped to improve their pedagogy by improving their teaching skills (Appendix 17). Thus, the use of ICT technologies helped in developing suitable learning opportunities for the students (T3, Appendix 17).

When demonstrating complex diagrams and key concepts in mathematics T3 (Appendix 16), indicates that ICT tools can promote visual understanding of key concepts. Added to the enhancement of learning is the effective use of ICT tools to increase attainment in processing information, concept mapping and modelling. Online dictionaries and 'edublogs' also help to increase the understanding of key words in the English vocabulary of learners when utilised by teachers (T7 and T10).

When answering the question on the benefits of integrating ICT pedagogy, most teachers related their answers to students' attainment levels. The use of ICT helps to meet the requirements of different learning styles, which makes it possible to achieve higher academic attainments (T2, Appendix 16). Most teachers acknowledged that the integration of ICT has worked positively to close the learning gaps and raise the attainment of pupils in their departments. Further, all teachers agreed that the use of ICT enhances learning through independent work. T5 commented that 'learning achievement goes up when teachers allowed differentiated independent learning' (T5, Appendix 19). Through independent learning, guarded questions are offered to students to improve their learning.

Most teachers also believed that the use of ICT can accelerate learning by ensuring adequate work to support both the weaker and more able students in class. According to T5, the use of ICT has the ability to 'offer a greater variety' of work to learners (Appendix 19). ICT has the added advantage of extending the 'breadths and depths of interaction' of all the learners (T5,

Appendix 19). T3 believed that ICT helps students to complete differential tasks within the school and helps teachers to provide quick feedback on completed work to improve students' attainment. However, T6 believed that there is little evidence of the impact of ICT on pupil attainment, and that ICT is not necessary for higher learning gains in studying mathematics. Refuting this claim, T5 agreed that there is a significant impact of ICT on pupils' attainment (T5, Appendix 19) when used consistently to complete both schoolwork and homework. Most teachers commented that negotiation of learning outcomes is better with both students and teachers when ICT strategies are effectively incorporated into pedagogy.

4.7.2 Part 2/Code 2: Barriers to ICT pedagogy

Interview question 2: What are the barriers to integrating ICT in mathematics education?

In answering this particular question, the teachers mentioned several obstacles or barriers that are creating problems in integrating ICT pedagogy into the teaching of mathematics in secondary schools. Particularly for T3 and T1, the main barrier to the integration of ICT in the pedagogy of the departments is hardware problems. Most teachers said that the lack of proper maintenance of ICT tools such as computers and interactive white boards discourages them from using ICT (T3, Appendix 17; T1, Appendix 15). They further mentioned that in most secondary schools, there are insufficient numbers of computers to enable student access, with limited computer suites and long waiting times for booking. Schools often need to work with out-of-date ICT tools that are slow and inefficient. Very slow broadband with poor networking facilities also makes the internet connection in schools very slow, which can make using online resources frustrating.

The absence of 'curriculum, resources and software problems' was reported to be a major barrier to using ICT integration (T4, Appendix 18; T5, Appendix 19). The inability to effectively deploy ICT in their lessons was attributed to poor ICT training on using software applications and time wasted looking for the appropriate resources. This has greatly affected their confidence and discouraged them from using ICT. The absence of experience in using ICT due to lack of confidence and skills has made most teachers resist ICT integration. This attitude of teachers has led to unsuccessful ICT integration in some schools. In addition, due to the negative perceptions and conservative attitudes of some teachers in adopting modern teaching methods, the lack of interest in using ICT was reported especially among teachers who were in their 40s (T2, Appendix 16; T4, Appendix 18; T6; T8).

T1 and T5 mentioned that lack of adequate skills and times are other obstacles to ICT integration (T1, Appendix 15; T5, Appendix 19). This was agreed by T3, who stated that the lack of 'teachers' expertise' remains the greatest barrier in his school (T3, Appendix 17). In addition, the inability to have the time to train due to heavy school workloads worsens the situation (T5, Appendix 19). Restrictions of the national curriculum were also mentioned as barriers to integrating ICT. T5 stated that the time and space available on the school's timetable for the coverage of the entire subject curriculum and the lack of interest of the school teachers make it difficult to integrate ICT into his secondary school's pedagogy (T5, Appendix 19).

Prior ICT experience was reported as a significant obstacle to integrating ICT into teachers' pedagogy; for example, T1 refused to use ICT due to the lack of 'past experience' in using computers (T1, Appendix 15). Supporting this claim, T4 mentioned that although several attempts had been made to include ICT in his pedagogy, he continues to be reluctant and remains uninterested due to personal ICT experiences in the past resulting from poor ICT exposure in teacher training institutions (T4, Appendix 18). Meanwhile, T2 could not integrate the ICT properly into her pedagogy due to the belief that ICT applications were difficult to learn (T8).

In this section of the interview, most of the teachers mentioned that the lack of sufficient knowledge regarding the use of ICT tools has created problems in integrating ICT pedagogy in their departments. Compounding this problem is the lack of knowledge on using up-to-date software (T2, Appendix 16). Most teachers also indicated that lack of available support staff to provide the needed knowledge prevents them from using ICT (T2, Appendix 16). According to T3, (Appendix 17), not knowing whom to approach for assistance is the greatest barrier to using ICT. Most teachers lamented the absence of whole-school initiatives to provide technical knowledge of ICT tools and applications when they are in difficulty, which further discourages them from using it (T5, Appendix 19). Nevertheless, T5 reiterated that 'having champions within the department matters most and teachers are not expected to know everything as a leader' (T5, Appendix 19). He believed that 'a greater opportunity to be creative' is vital in leading a department to integrate any educational initiative (T5, Appendix 19).

4.7.3 Part 3/Code 3: Facilitators of ICT pedagogy

Interview question 3: What are the factors that will facilitate ICT in Mathematics education?

In response to this particular question, teachers commented on the different training and development strategies that enable the use of ICT. Also mentioned were the existing tools available to promote ICT. All of the research participants stated that they use several tools and applications to facilitate ICT in their pedagogy. Three teachers in particular identified their ICT facilitators as 'visual and dynamic aids' (T1, Appendix 15; T4, Appendix 18; T5 Appendix 19). The tools discussed included 'videos and films, interactive boards, YouTubes, tablets' (T1, Appendix 15 and T4, Appendix 18), 'educational websites and online applications including calculators' (T2, Appendix 16), and 'WhatsApp, Twitter, [and] mobile phones' (T3, Appendix 17).

According to L1, the integration of ICT tools allows learners to develop effective analytical skills and problem-solving skills. In addition, the development of ICT tools has helped students in the development of the conceptual understanding of mathematics (T6 and T7). T6 and T7 agreed with other teachers on using ICT tools such as tablets, whiteboards, and internet applications to improve learning. The Mathematics teachers mentioned that they were using the interactive whiteboard and many different online applications to facilitate ICT in their pedagogy (T2, Appendix 16; T4, Appendix 18).

The interviews demonstrated that in most cases, the lack of effective ICT training makes the teachers technologically handicapped, and thus the usefulness of ICT tools is not fully recognised in mathematics education. Notably, the teachers found the provision of training to facilitate the use of ICT pedagogy (T3, Appendix, 17). Both T2 and T7 were against any facilitation of ICT received externally (Appendix 16) and indicated that the ongoing in-service training equips teachers with the right skills to implement ICT pedagogy. This is because T2 believed that the lack of continuous training in the past by external training providers led to less efficient ICT integration (T2, Appendix 16). T6 and T8 pointed out that if the schools' administration involved more experienced and knowledgeable local teachers, then they could deal with the current situation of integration in a better way. Recruiting a technology co-ordinator was therefore thought to be very important for the integration of ICT in the secondary schools' pedagogy (T4, Appendix 18).

In relation to training opportunities, the teachers discussed several professional on-going developments within their schools. T2 and T3 mentioned that both the departmental training sessions and the training sessions provided by the school impacts on their ICT pedagogy (T2, Appendix 16; T3, Appendix 17). Both T2 and T3 commented that their school's professional development activities include working in teams to support all (Appendix 16; Appendix 17). However, schools may be reluctant to provide continuous professional development due to other school priorities (T1, Appendix 15). The need to provide training sessions through INSET/staff training and sharing good practice was seen as vital to support teachers to integrate ICT (T3, Appendix 17).

4.8 Teachers: Questionnaires

The data was collected from 40 out of 50 teachers using interviews and Questionnaires from 10 schools in south London. The responded represented 80% of the target group. The following represent the analysis of the data with the findings presented according to the objectives in the following sections.

The following data was obtain using SPSS tool to analyse the questionnaires

Benefits of integrating ICT pedagogy;

- Sustained student interest:. 55% Agreed
- Aid lesson planning with wide range of resources: 75% Agreed
- Improved the attainment of children: 60% Agreed
- Enhance interactivity and exploration: 75% Agreed

Barriers to integrating ICT pedagogy;

- Lack of curriculum resources: 66% Agreed
- Software and Hardware problems: 32% Agreed

- Lack of experience with ICT tools: 35% Agreed
- Negative attitudes: 40% Agreed

Facilitators of integrating ICT pedagogy;

- The Availability of ICT tools: 65% Agreed
- Experience and personal knowledge of using ICT: 85% Agreed
- School leadership: 75% Agreed
- Opportunities for training: 26% Agreed
- The following table shows the full summary results.

Table 4: Summary data analysis using SPSS

<table>
<tr><td>Research questions</td><td>Interview questions</td><td>Strongly Agreed (SA) Agreed (A) Disagreed (D) Strongly Disagreed(SD)</td></tr>
<tr><td>Benefits of integrating ICT pedagogy</td><td>Sustained student interest.
<table><tr><td>SA</td><td>A</td><td>D</td><td>SD</td><td>Total</td></tr><tr><td>30%</td><td>55%</td><td>10%</td><td>5%</td><td>100%</td></tr></table>
Aid lesson planning with wide range of resources.
<table><tr><td>SA</td><td>A</td><td>D</td><td>SD</td><td>Total</td></tr><tr><td>20%</td><td>75%</td><td>5%</td><td>0</td><td>100%</td></tr></table></td><td>Improved the attainment of children.
<table><tr><td>SA</td><td>A</td><td>D</td><td>SD</td><td>Total</td></tr><tr><td>25%</td><td>60%</td><td>10%</td><td>5%</td><td>100%</td></tr></table>
Enhance interactivity and exploration.
<table><tr><td>SA</td><td>A</td><td>D</td><td>SD</td><td>Total</td></tr><tr><td>20%</td><td>75%</td><td>5%</td><td>0</td><td>100%</td></tr></table></td></tr>
<tr><td>Barriers to integrating ICT pedagogy</td><td>Lack of curriculum resources
<table><tr><td>SA</td><td>A</td><td>D</td><td>SD</td><td>Total</td></tr><tr><td>20%</td><td>66%</td><td>10%</td><td>4%</td><td>100%</td></tr></table>
Software and Hardware problems
<table><tr><td>SA</td><td>A</td><td>D</td><td>SD</td><td>Total</td></tr><tr><td>68%</td><td>32%</td><td>0</td><td>0</td><td>100%</td></tr></table></td><td>Lack of experience with ICT tools
<table><tr><td>SA</td><td>A</td><td>D</td><td>SD</td><td>Total</td></tr><tr><td>50%</td><td>35%</td><td>10%</td><td>5%</td><td>100%</td></tr></table>
Negative attitudes.
<table><tr><td>SA</td><td>A</td><td>D</td><td>SD</td><td>Total</td></tr><tr><td>60%</td><td>40%</td><td>0</td><td>0</td><td>100%</td></tr></table></td></tr>
</table>

Facilitators of integrating ICT pedagogy	Factor	SA	A	D	SD	Total
	The Availability of ICT tools	30%	65%	5%	0	100%
	School leadership	15%	75%	10	0	100%
	Experience and personal knowledge of using ICT	10%	85%	5%	0	100%
	Opportunities for training	70%	26%	4%	0	100%

4.9 Chapter summary

The responses of teachers to the interview questions and questionnaire revealed a common trend. The emerging themes on ICT-integrated pedagogy demonstrated that teachers held contrasting views on the benefits and obstacles in ICT integration. In terms of the perceptions of teachers' role in ICT, most teachers lamented the lack of adequate ongoing training to promote ICT integration. Additionally, the lack of technical support, lack of accessibility of ICT tools, and lack of effective training makes it difficult for teachers to deploy any effective strategies for using ICT to change pedagogical practices.

The next chapter discusses the findings from the data collected from the participants and how these relate to the findings of the literature review. All associated theories between the findings and the research are discussed.

CHAPTER 5: EVALUATIONS AND FINDINGS

5.1 Introduction

The aim of the research was to find out the impact of ICT on mathematics education? To be able conduct the research detailed, I used own teaching experience through case studies and the responses from both students and teachers to answer the following research questions;

- What are the benefits of ICT in Mathematics education?
- What are the barriers to ICT in mathematics education?
- What are the factors that will facilitate ICT in Mathematics education?

The following sections will discuss the findings from the case studies and the responses from both teachers on the impact of ICT in mathematics education. First, the reasons for selecting GeoGebra as a tool for the ICT lesson will be discussed followed by the case studies and the interviews from teachers.

5.2 Rationale for the choice of GeoGebra

GeoGebra was use as ICT tool due to its ability to enhanced understanding through the multiple representations of views and its user friendly nature. In Singapore, Venkataraman (2012) conducted a study in secondary mathematics examining inventive activities used to develop geometrical thinking ability in GeoGebra. The study indicated that students who studied through the software recorded significant progress in understanding mathematics, a basis for additional deductive reasoning in mathematics (Ritz, 2009). The research concluded that the ability to use the dragging feature in GeoGebra was the vital component that impacts the generalisation achieved by students using the program to learn, since abstract concepts are made more meaningful through visualisation (Venkataraman, 2012). The GeoGebra software provides learners with the interactive, reasoning and visualisation (Hohenwarter, 2007) necessary to understand key concept. The tool offers the teachers the benefit of inviting learners to contribute to a compelling investigation through affirming the fundamental computational and mathematical essentials (Clements, 2004).

Using GeoGebra to teach mathematics is more effective than using other software because it has the ability to provide learners with deeper understanding of the key concepts (Chimuka, 2017). Again the use of GeoGebra allows students to identify and explore different theorems through the process of 'trial and improvement' (Ruthven and Hennessy, 2003). The pedagogy in mathematics using GeoGebra has the additional ability to empower students to begin with the process of discovering new problem-solving skills, which aids the understanding of a mathematical conjecture (Ritz, 2009). According to Chimuka (2017), using GeoGebra enables students to perform experiments and make discoveries that enhance students reasoning skills in a way that would be impossible through 'chalk and talk' instruction.

5.3 Lesson design

A lesson design simply refers to the strategy employed by the instructor to structure, develop and implement a lesson plan. The effectiveness of a lesson design is characterised by the following features. First, the learning aims and objectives should be known at the start of the lesson. The activities were centred on the learning outcomes. In the lesson design, clear steps were set on the consolidation of pupils understanding in the previous lesson before the introduction of new concepts. The design of the lesson took into consideration the provision for students to be allowed to work in teams or pairs. The importance of this is to enhance their understanding by providing them with the chance to support their peers to understand and develop knowledge in ways in which the teacher cannot assist.

Contrary to these good designs, poorly designed lessons tend to be fully instructor-directed and teacher focused. They fail to avail of the opportunities for the exploring of knowledge prior to the provision of the instructions. A well-designed lesson usually encourages the instructor to begin the lesson by determining how much knowledge of the lesson's topic the students have. This may be through group discussions or a report out of the class action. Also, it allows the teacher to pause in breaks to ask questions and allow the learners to express how much of the lesson they have understood.

5.4 Frame work for ICT lesson

Using the TPACK and the Knowledge Quartet framework for the lesson design necessitated an exploration of the prior knowledge of the students and, as such, encouraged the students' input. The lesson planning is viewed through the Technology, Pedagogy, and Content Knowledge (TPACK) framework). By using the TPACK framework, a teacher is able to draw attention to the diverse kinds of information needed for effective integration of a digital technology (DT) (Shulman, 1986). The TPACK framework incorporates technology and teaching in a desired method by providing the interconnection between three categories of knowledge delivery that combine for fruitful digital incorporation (Koehler & Mishra, 2009; Cox & Graham, 2009).

The TPACK framework was considered against other modules such as Maslow's Hierarchy of Needs or Bloom's Taxonomy modular because it affirms technology, content and pedagogy to be the key advocators of education whilst the former modular explains and shapes the dynamics of learning processes (Archambault & Barnett, 2010). Notwithstanding the number of challenging assumptions of using the TPACK framework, for example, the assumption on secluded components being understood as separate overlying circles representing empirical reality is absurd. Also, the supremacy of content knowledge being considered as one of the central educational apprehensions and the unrealistic assumption of the model without drivers and goals makes TPACK student-centred framework, a worry for many educators (Clark-Wilson, et al., 2014).

I evaluated my expertise in line with the TPACK and PCK frameworks. As part of my planning for TCK, integrating GeoGebra in my lesson required a detailed understanding of how the various circle theorems were going to be introduced. For example, the need for calculating angles and descriptions of the various theorems were identified as part of my TCK expectations. The use of TPK helped me to be mindful of my role as a teacher and transmitter of knowledge. Using this role allows teachers to correct errors or misconceptions in the learning process (Hadjidemetriou & Williams, 2002). My planning for PK and PCK enabled students to carefully be guided to use an accurate collective dialogue.

5.5 Lesson plans

A lesson plan is a description, a thorough learning trajectory of a sequence or order of instruction. The learning trajectories entail the learning goals and expectations, the lesson developmental path and the set of activities matching the thinking levels in the developmental path, which enable the learners to develop higher-order thinking. A lesson plan acts as the blueprint and guides for a lesson including the teaching activities, classroom arrangement, student engagement and the systematic approach of teaching the concepts.

5.5.1 Experience of teacher with ICT

To be able to plan the lesson effectively, Rohaan et al. (2010) argue that the teacher's self-efficacy with digital technology should be examined against their own practice. A well-developed lesson plan incorporates the best practices and correlates with the teacher's philosophy of education. Rohaan et al. (2010) indicate that when planning lessons, teachers should be mindful of their self-efficacy in connection with their capabilities with ICT. By taking into consideration of own expertise in using ICT, acknowledging the knowledge gaps and taking steps to resolve them boost the confidence with the application in the lesson.

5.5.2 Objectives of the lesson

The aim was to design a lesson and teach circle theorems using GeoGebra to facilitate students' mathematical reasoning. The topic was taken due to the difficulty of teachers to demonstrate the hidden views of the shape when using pen and paper. My aim was to develop activities that would be effective for all pupils, with the goal to inspire students to learn. Also vital was the aim to use digital pedagogies aligned with the learning goals that students already demonstrated throughout their own learning: interaction, engagement, and peer-to-peer discussions with focused assessment. In peer-to-peer learning, for instance, the aim was for me to promote students' reflections and logical reasoning with comments on peer discussions and student ownership of the learning through student-centred activities.

5.4.3 Consulted wide range of resources

To be able to teach the lesson successfully, I consulted variety of resources on the topic to understand how other teachers have taught similar lessons, since teachers' approaches to the incorporation of ICT have shaped the foundation of how they have advanced their pedagogy (Saudelli & Ciampa, 2016). To overcome my anticipated difficulties in incorporating ICT in the lesson, Saudelli and Ciampa (2016) cites that teacher's beliefs and confidence are influenced by their attitude and the chosen resources towards technology integration. In addition, I was mindful that connectivity among people, according to Siemens (2006), has the ability to transform the way that people access, interact, and process knowledge. Therefore, thought was given to how to plan effectively using numerous activities that maintain full engagement with the learning. By deploying technology as a servant, teachers are able to advance the reasoning and conjecturing in mathematics education (Goos, 2014).

5.4.4 Experience of students with technology

According to Halldórsson et al., (2009) and Murray (2016), knowing pupils' backgrounds knowledge in the topic in the lesson planning, including skills and experiences, ensures a better outcome of the lesson. The Knowledge of pupils' background prior understanding in the topic helped me to plan an inclusive lesson in view of students' different past experiences with ICT teaching (Godwin & Sutherland, 2007). When teaching difficult concepts, Ruthven et al., (2008) advocate the using of technology as a vehicle to break down key contents through a structured approach. To support students who had little experience with the tool, I planned to use scaffolding task activities. The use of scaffolding actives ensures step-by-step delivery of the key concepts to promote the development of the required technological skills (Ruthven, 2014 & Murray, 1996).

5.5 Case study 1: lesson on Circle theorem

5.5.1 Review previous lesson

A good place to start when teaching a lesson in circle theorems was to deliver a recap of angle facts relating to circles. I aimed to ask students to do some exploration into angles in circles using pen and paper. The aim was to help the students to apply their acquired knowledge and skills in previous lessons in various situations (Malibar and Pountney, 2002). For instance, the concept of calculating angles in the previous pen-and-paper lesson encouraged a feeling of conjecturing and improving on the answers to the task. After that students were given more practice, starting with simple cases and then moving on to more demanding questions involving more than the use of one theorem.

5.5.2 Misconceptions were corrected

During the lesson, time was allocated to discuss pupils' completed worksheet with feedback. This role enabled me to gain full control of the learning process, as I had the opportunity to help students to overcome their difficulties and rectify misconceptions resulting possibly from the integration of the technology. According to Becta ICT Research (2003) since computer assisted learning is individualistic it is easy for a student to face difficulties due to knowledge gaps. Also, a student is at a loss of the expectations during an ICT lesson if instructions were not carefully followed and understood. This frustration is compounded when the ICT lessons are held at a different environment from the usual classroom since the alternation of rooms creates discontinuity of knowledge. This is because often teachers uses the ICT lessons as a continuation of the previous traditional lesson and the lessons became more beneficial to students who have already understood the concepts from the previous lesson.

5.5.3 Access to online resources

Each activity was hyperlinked to the online resources (www.mymaths.co.uk) for students to gain more practice. The use of thorough supervision of students' work in progress was required to minimise disengagement (Jones et al., 2009) and enabled me to provide directions to resolve misconceptions. To be able to accommodate for the different learning needs in the class, differentiated task were planed and online resources were used to extend pupils learning. Also, where pupils needed tailored support, to promote classroom engagement, ready-made resources were provided (Appendix 3).

5.5.4 Collective learning

I planned whole-class discussions as part of the lesson. According to Godwin and Sutherland (2007), embarking on whole-class discussions is essential in mathematics learning, as they enable collective student knowledge building. Although the whole class discussions gave all students the opportunity to receive collective feedback on their progress in the lesson, I physically moved around the class to provide individual tailored feedback and supported each student in difficulty, recognising the different needs of each student (Saudelli & Ciampa, 2016).

5.5.5 Facilitated assessment

To prompt students to make simplifications and interpretation on calculated answers, worksheets were designed to request the reason for any decision taken and to discourage generalisations (Mavrikis et al., 2013). Also, I endeavoured to caution pupils to critically scrutinise their findings and not recklessly accepted their answers from the use of ICT as their default final answers (Lu, 2008). The use of 'Why' and 'what' questions such as, "What happens to the size of angle after varying the size of the circle?" were structured into the lesson to motivate learners to remain focus on task (Appendix3).

5.5.6 Increased learning confidence

My supervisory role ensured that children were confident to think through every stage carefully through justification of proof. This student-focused, inquest-based approach to teaching with ICT (Dochy et al., 2003; Harada et al., 2008) promotes a great degree of student partnership and application of reasoning and thoughtful skills when ICT are used as mechanism for teaching (Harada et al., 2008). Once students were familiar with the vocabulary, they were able to use the right terminology and language to justify their reasoning behind the calculated results. The use of PCK demonstrated that unfamiliar vocabulary use of ICT in lessons was an obstacle to the understanding of mathematical concepts (Hadjidemetriou and Williams, 2002; Jones et al., 2009).

5.5.7 Exploration

Also, at the start of the lesson, time was allocated for exploratory-based learning (Malibar & Pountney, 2002) to offer students the chance to freely experiment and justify their taught process. This made it possible me to partner with the use of technology to provide learners with innovative motivation beyond their learning. This approach enabled learners to develop a critical thinking on key concepts (Elliot, 1998; Zbiek et al., 2007; Goos, 2014).The technological tool aided the exploration of a set of circle theorems through dragging of the points on the circumference of the circle to discover the connection between the changes in the size of the angles. For example, the theorem that the ‘angle at the circumference is half the angle at the centre’ is difficult to understand when using pen and paper, since most students find it difficult to realise that there could be four other cases (Appendix 3) to consider.

The use of the technology aided the design of the lessons in using strategies that enabled me to explore the key concepts of the topic in a step by step manner. Exploring one concept at a time when learning with ICT helps to structure the knowledge (Ruthven et al., 2008), which is vital to accelerate understanding of difficult concepts in mathematics education. The lessons were structured to allow the exploring of one concept at a time. Such an approach allowed me to use the technology as a servant (Goos, 2014).

5.6 Lesson report

5.6.1 Task 1: The angle at the centre of a circle is twice the angle at the circumference of the circle from the same arc.

The test of this theorem consisted of several parts. Initially, all students (Groups A and B) arrived at a construction that demonstrated the inscribed angles and the angles at the centre intercepting at the same arc. Students from Group A quickly noticed that the degree of the angle at the centre appeared twice the degree of the angle at the circumference. Meanwhile, students from Group B varied the calculations, even though it was obvious by using the input function to compute the ratio of the degree of the angle that appeared at the centre to the degree of the angle at the circumference (entering β / α, which showed f = 2 in Algebra view). My role guided the students on the best choice of opportunities to arrive at the answer when faced with numerous ICT options. In the opinion of Monaghan (2004), teachers' role as facilitator of ICT in mathematical knowledge when guided prompts were used assisted students with their discoveries. Although this views as indicated by Sherman (2014), rather creates source of inconsistency in the general responsibilities of the teacher. This is because the teacher becomes the centre of mathematical authority and reasoning to a catalyst of architect of students' mathematical knowledge and activity. Upon performing the drag test (Finzer & Bennett, 1995), S4 stressed the several ways to visualise the inscribed angle theorem (figure 1b) while confirming what he was familiar with in the previous lesson when mymaths.co.uk was used.

5.6.2 Task 2: Angles inscribed in a semicircle are right angles

According to Saudelli and Ciampa (2016), understanding the TPACK framework equips teachers to find ways that promote learners to experiment an institutional perspective in a mathematising process. Most students started with just the drawing of the circle, preceded by exploring the drawing of a line passing through the centre of the circle. This was followed by the construction of two chords to produce the inscribed angle. As seen in figure 2a, the angle produced between the chords was 90°. The understanding of the concept was affirmed when S2 dragged the corner of the inscribed angle along to the other semicircle. In figure 2b (Appendix 3), the resultant answer was 270°, as a result of the dragging, showing that the angle of interest was measured as 90°. According to Finzer and Bennett (1995), the use of dragging to explore the changes in the size of angles aids students understanding in circle theorem.

As specified by Blum and Niss (1991), understanding ICT barriers that could affect the different stages of the modelling processes is vital to classroom practice. The barriers identified included the slow rate at which the software was functioning perhaps due to the over reliance of the internet facilities. As a consequence, this effect influences the level of the mathematising process and often hinders learners' perceptions of tasks and the development of modelling competencies (Blum & Niss, 1991). Once the barriers were resolved, S4 and S4 constructed the inscribed angles and extended the triangle outside the circle to investigate the link between the angle externally and the angle internally. S4, S3 mentioned, "If the central angle is a straight angle then that means that the angle at the centre and the two inscribed angles added up must add up to 180 degrees" (figure 2c Appendix 3).

5.6.3 Task 3: Angles in the same segment are equal

As indicated by Bosch et al. (2006), the act of solving mathematical problems consists of both the knowing how to do it and knowing why to do it. Doerr and Pratt (2008) differentiate between exploratory models (expert, teacher) and expressive models (learner, students). Most students were successful in constructing two inscribed angles that met at the same arc. This was interesting to notice, since I was expecting students to construct another inscribed angle by deploying the drag function. However, the Group B students further demonstrated the construction, as illustrated in figure 3a, through the drag test (Finzer & Bennett, 1995) using the already constructed inscribed angles.

Zbiek et al., (2007) indicated that students engage in exploratory activity when given a pre-procedure to carry out. When I probed S5 and S6 why the need for dragging the apex of the inscribed angle? Both S5 and S6 demonstrated expressive activities to affirm on why they decided on preferred approaches to justify their choices. They both understood the procedures to use through exploration using the technology without my direct interference in the required steps to take. Furthermore, for example, S5 responded "We're testing to see if it's actually true that any angle at the circumference are equal, so not just those specific angles, but any angle at the circumference will be equal". S2 also indicated this theorem's association with the Inscribed Angle (Task 1, Appendix 3) Theorem and suggested measuring the central angle. The students in Group B recognised the connection between this theorem and the

Inscribed Angle Theorem by entering β / α, to obtain f = 2 as seen in the Algebra Side bar and typing β + α= 360 degrees, as seen in figure 3c Appendix 3).

5.6.4 Task 4: Cyclic Quadrilateral Theorem

The awareness of the knowledge of dragging shapes in dynamic geometry (Finzer & Bennett, 1995) allowed me to guide students cautiously on how to carefully drag a shape to preserve its properties. For example, with the aid of dragging the shapes, the class was able to visualise the nature of the change in angles and through investigation confirm that opposite angles of a cyclic quadrilateral add up to 180^{O}. The Cyclic Quadrilateral Theorem raised an important discussion on construction for the Group A students who used the input bar and confirmed that the opposite angles of the cyclic quadrilateral were supplementary.

Most students started by constructing a circle and finding their quadrilaterals by indicating four random points on their drawn circle and joining them through line segments. S2 clarified the results by measuring the angles; S1 confirmed it again by entering the total of the opposite angles (α + β = 180° in the Algebra view of the input bar. The peer-to-peer discussion that emerged made the students understand and affirmed that the construction of the cyclic quadrilateral was correct (figure 4a, Appendix 3). S2 noticed that the total of the opposite sides would be wrong if the points were not indicated on the circle when dragged (Finzer and Bennett, 1995) out of the circle (figure 4c). S3 quickly managed to fix it to ensure that the total of the opposite angles added up to 180 degrees. S4 was able to affirm through the dragging test (Borwein, 2005; Ruthven, 2009) that the opposite angles were always supplementary. The ensuing discussion on the activity triggered a lot of enthusiasm among the students.

5.6.5 Conclusion

Pedagogies with ICT have the benefit of high achievements in mathematics education through motivation, concept formation, high-order thinking, visualisation, analysis, abstraction, comprehension and retention, and rigour. Students' feedback and interviews revealed that lessons delivered with digital tools have the ability to transform learning.

When seen through the lens of the TPACK, the lesson demonstrated the need to carefully think through the various kinds of knowledge in the planning of ICT-integrated learning for a successful lesson. I believe the incorporation of GeoGebra, an ICT tool helped to create a new environment not usually experienced in lessons. Most students demonstrated high enthusiasm about using the software to advance their understanding of key mathematical concepts. As such, using ICT continues to justify the understanding of key mathematics concepts, providing the needed intuition, insight, and exploration of similar but new patterns and relationships (Borwein, 2005; Ruthven, 2009).

Although the study established that there was a significant gain in learning when ICT was incorporated in the learning of mathematics, existing research on using integrated technologies for teaching mathematics has indicated the need to scrutinise the professional learning that complements the use of new technologies into teaching practice. Further acknowledged in the literature is the need for pedagogies that promote integrated technology mathematics education that requires an alteration in teachers' professional attitude taking into account the barriers of learning (Gueudet & Trouche, 2009: Ruthven, 2009).

5.7 Case study 2: Lesson in Linear functions

5.7.1 Introduction

The reason for the lesson again was to investigate on the impact of ICT pedagogy in linear functions. The relationship between dependent and independent variables in linear functions and the impact they have on each other is difficult to observe by students when learning with pen and paper hence the ICT lesson. The lessons were categorised into two lesson episodes, to answer the research questions whether, the use of an ICT has any impact on mathematics education?

To achieve this aim the lesson plan ensured the design of learning that introduced the most basic concepts to the complex. I also examined the findings of the general design of these lessons. The main aims of the lessons are as follows:

- Introduction lesson to linear functions using GeoGebra
- To sketch a linear graph when given the gradient and point of interception
- To construct a table of values for given linear functions and draw a linear graph

5.7.2 Task 1: Introduction lesson to linear functions using GeoGebra

The main objective of the lesson was to introduce students on how to use GeoGebra for graphing. The lesson was an introduction lesson, which introduces both the software GeoGebra and the topic of graphing. The introductory remarks clearly express why the employment of the chosen graphing software is important instead of using paper. Remarkably, in the first minute, I expressed where a student can get the software to be used at a personal level. At the start of the lesson, students were asked to open the software after explaining and showing the class the location of the Geogebra.org software on the school's system. I further demonstrated to the class how to use the functions on the software and what they are for. I further explained how to access the graph functions in particular and then went ahead to explain how a learner can access the different features to manipulate the drawn graph, such as the use of cursor to access different points of the graph and to show a magnifier to increase the visibility. To understand the right level to pitch the lesson, I used a range of strategies, including open-ended questions to check with pupils' prior understanding in functions. After a group practice on the initial application on the use of the software, an independent practice session was given to the learners so that they could execute the skills learnt on their own. The task provided took account of what students already knew about linear functions. It also involved the assessment of the ability of the students to understand and use the basic functions of the GeoGebra software to draw graphs.

5.7.3 Task 2: To sketch a linear graph when given the gradient and point of interception

The objective of the practice exercise was to understand how to graph a line function using GeoGebra. Most students were able to use coordinates to sketch a linear graph with no difficulties. However, few students had difficulties when they were presented with only the gradient and asked to draw a linear function. After dialoguing with the students and inviting feedback from completed students, all the students were now in a position to complete the assigned task. Asking students to use GeoGebra to draw a function given the point of intersection was again not answered correctly by most students because most students failed to visualise that although the gradient of functions can be different, the point of intersection and the gradient are used to find the equation of the function. However, with the gradient and the y intercept, most students were able to overcome their difficulties and were able to draw the line with no difficulties.

5.7.4 Task 3: To construct a table of values for given linear functions and draw a linear graph

Throughout the lesson, the use of the ICT supported pupils in understanding the mathematical experimentation in line graphs in the form $y = x + c$. The use of the software provides encouragement to students learning through trail and improvement, which played a critical role in the understanding of the topic. Also, most students were able to engage with the mathematical speculations and experiment to enhance on their knowledge in the subject.

Immediately past this point, the students were introduced to the following linear equations:

$$Y=2x-7 \text{ and } Y=-x+5.$$

Past this point, the class was introduced to how to use the graphical feature to plot these two equations. Prior to introducing other actions and equations, I assisted the students to erase a graph when a mistake was made. Using prompts, I used examples to explain to the students how the gradient of the linear functions can be determined. I then gave the students the option to create problems and try to solve them with the software.

5.7.5 Lesson report

Most students were able to construct a table of values given the linear functions. Few students were able to link the skills gained in completing this task to a similar task in Excel. The transfer of knowledge in completing this task was not obvious because of the lack of understanding of using GeoGebra to draw a table. Most students were able to draw the linear graph given the function.

Revisiting a lesson with the aim of improving future lessons is crucial for an instructor. More so, when teaching resources are involved, reflection is vital for ensuring that the teacher employs the resources in a more effective manner. The use of technology in linear functions includes instructing and picking up, managing new types of dynamic portrayal and correspondence. However, it is likewise certain that the requirement for suitable expert advancement to help educators in planning innovation upheld exercises remains fundamental. Exclusively giving innovation is lacking to the effective joining of new unique devices to instructing.

As Olsson (2017) reports, giving access to learning software do not really prompt innovation. However, there is confirmation that proper expert development of ICT and collegial help can help instructors' readiness to incorporate innovation into their educating and can boost their ability to create fruitful innovation (Öçal, 2017). Some portion of this may involve helping educators in understanding the affordances, limitations, and the general instructive nature of new authentic assets in connection to the particular points in school mathematics education (Mata-Pereira & Ponte, 2017).

5.7.6 Pupil's response

The analysis on the semi-structured interviews with students, on the using of dynamic software (GeoGebra) to teach linear functions, revealed that learners were profoundly motivated to learn. Students (S1 and S4, Appendix 7) reiterated that using the GeoGebra tool for learning linear functions made their learning fun and exciting. Students (S2, S3 and S5, Appendix 7) supported the initial views on motivation and commented that using GeoGebra provided them with the necessary encouragement to keep on working through the process of trial and improvement even when their answers were wrong. Student S3 said, 'I would have quit trying for the right answer if it was investigation using pencil and paper due to cancelling my work and starting on a new page'. The ability when using GeoGebra to provide learners with a clean page to work on when mistakes are done is an incentive to encourage the completion of more work.

Additionally, as indicated by the interviews, most students reported that using GeoGebra for learning was reasonable and simple as far as drawing geometrical shapes. Most students commented that I was able to support their learning through the timely feedback presentation on the completed work. Also, most students were quick to comment on the sequence of the lessons with the differentiated work. According to students (S2 and S5, Appendix 7), the lesson was set at the right pace, which enabled them to remain engaged in the learning. The visualisation and the capability to use the multiple representation views function in GeoGebra were commented on by most students as the key feature that enabled understanding to take place.

5.7.7 Impact on learning functions with ICT on mathematics

The students reported that GeoGebra software had a positive impact on their mathematical performance. In spite of the fact that the post-execution test scores demonstrate abatement contrasted with the pre-accomplishment test, the study demonstrates that there are noteworthy contrasts between the post-execution tests on utilising GeoGebra. The study demonstrates that the utilisation of GeoGebra in the learning can give a significant effect in enhancing the learner's ability. Similar to a study done by Masri et al., (2017), the consequences of this investigation additionally demonstrate that the utilisation of GeoGebra in the learning and teaching has the ability to improve learners understanding and mathematical reasoning capacity. In the same study, it was reported that students who used GeoGebra scored higher compared to those who learned with customary strategies. In view of these findings, GeoGebra is extremely useful in the conventional classroom and more powerful than customary instructing. The increase in learners' accomplishments' tests scores is likely because of variables of their fascination towards the dynamic technology. The advancement of innovation instruments expands students' enthusiasm to make sense of the new thing. The learners tend to investigate the technology to apply in learning mathematics.

The contrast in the performance of learners using GeoGebra and those not using it demonstrates that utilising GeoGebra has a positive consequence on the student's accomplishment in mathematics education. A review by Leung (2017) likewise demonstrates that learners using GeoGebra outperformed those who did not use it. In light of their observations, this proposes that GeoGebra is exceptionally useful in the customary classroom educating scenario and more powerful than conventional instructing. The augmentation in the students' accomplishments' tests scores are likely because of variables of their fascination towards the innovation.

Also, similar to the findings of Jacinto and Carreira (2017), the learners' recognition was distinguished through an arrangement of a survey comprising nine things. The outcomes picked up from the survey demonstrate positive outcomes. The investigation found that the things in the poll that had the most reduced mean expressed that students can think innovatively and basically with a mean of 3.93, while the most elevated mean is 4.62, which is received for the first time demonstrates that the general pool of students concurred with positive explanations when ICT was used especially, GeoGebra.

The outcome of the study presumes that the utilising of GeoGebra in learning linear functions contributes to an advantage to the learners' certainty and their inspiration in learning mathematics. Hegedus et al., (2017) expressed that the utilisation of ICT in instructing and learning is not just to enhance understudy execution, but also inspiration, and stated that learners who reacted emphatically demonstrate the most elevated rate contrasted with different reactions. According to Hegedus et al., (2017), learners who used ICT for learning demonstrates enthusiasm for learning mathematics. In a similar research by Granberg and Olsson (2015), indicates that when ICT is used students gain comparable understanding through exploratory learning. It demonstrates that learning with dynamic technology could likewise trigger on-educational curiosity for learning mathematics.

To teach a lesson in linear function, the teacher's planning aimed at finding the resources to equip students with the skills of being able to identify the links between algebra symbols, tabular and graphical representation. In the planning to equip students, resources were designed to make students understand the different meaning of algebraic symbols. Undoubtedly, teaching linear functions is a complex procedure in which teachers plan to use diverse activities to help students to figure out the links that connect verbal, tabular, graphical and symbolic exemplifications. Using digital technology to help teacher establish this relationship is remarkable in mathematics education, even though the process of incorporating technology remains a complex and sensitive issue (Cuban, et al., 2001). The use of GeoGebra assists teachers to provide the clear impact of linear functions on graphs, visually for learners to think mathematically and appreciate the association between algebra and symbols.

However, it remains a controversial issue as to what role the integration of technology should contribute to students' mathematical reasoning and to what extent technology can enhance a teacher's mathematical pedagogy. Convincing enough, the use of the software made it easy to illustrate to students through dragging. Although it is possible for teachers to deliver lessons to make students understand algebraic functions without the use of technology, the presence of numerous challenges impeding students understanding is well known. The absence of technology makes it difficult to demonstrate the key concept of expressing relations, generalisations and formulae (Bell, 1996). Being aware of the student difficulties help in the planning of lessons that rely on technology to act as a medium to support students learning.

As commented on by many reviews, the learning of mathematics is centred on algebra and symbols rather than visuals. Hence, the use of technology has the ability to aid the visualisation that adds to the understanding of the subject. However, added to this benefit is the ability for students to understand the unique meaning attached to the symbols in functions when technology is used. In a digital environment, to generate a linear function, students are able to enter a formula and instantly produce a graph and a table. All three representations can be seen as a goal enhancing learners' mathematical experience. Through dragging, algebraic values in functions values in functions are changed, which enables teachers to be able to show learners the role of each symbol (Noss, 1997).

5.7.8 Conclusion

In most cases, the use of ICT allows students to own their learning without being greatly dependent on teachers, giving a feeling of accomplishment and excitement in having control and ownership of their own projects. The use ICT has the ability for teachers to enhance on learners understanding not usually possible through the traditional way of teaching. The use of ICT allowed students to critically solve the mathematical problems though exploration and discovery learning. With the appropriate design tools and tasks, the students felt the urge in seeking for deeper creativity to enhance their understanding in mathematics.

5.8. Research question 1: What are the benefits of ICT in Mathematics education?

This session examines the data generated through both the questionnaires and the semi-structured interviews on the views of teachers on the benefits of using of ICT in mathematics education.

5.8.1 Sustained student interest

The data analysis of the questionnaires indicated that although 85% of the teachers either strongly agreed or agreed, 15% either disagreed or strongly disagreed with the use of ICT to sustain student's interest in lessons. Also, the results of the interviews support the literature review finding that the benefits of ICT include sustained student interest (Al-Ansari, 2006; T1, Appendix 15). Ager (2000) similarly describes the benefit of ICT in motivating children through integrative applications. This agrees with the perceptions of Young (2014) and Phillips and Scrimshaw (1997), who indicate that using ICT tools promotes visual learning and enhanced excitement in learning. The belief that ICT can promote the advancement of talented pupils (OECD, 2011) was held by most teachers, who agreed that the use of ICT helps widen learner's accessibility to more educational resources to promote student creativity towards learning (Brown, 2011). The use of ICT has an added benefit of promoting excellent behaviour for learning since most students through the excitement of working with computer will remain fully engaged with their classwork(T4, Appendix 18)..

5.8.2 Aid lesson planning with wide range of resources

The data analysis of the questionnaires indicated that although 95% of the teachers either strongly agreed or agreed, 5% either disagreed or strongly disagreed with the use of ICT to aid lesson planning with wide range of resources. In addition, it was widely accepted by the teachers that using ICT allows them to plan lessons more easily and effectively (T4, Appendix 18). The teachers further agreed that the ability to use ICT to aid visual learners (T3, Appendix 17) ensures that teachers are able to plan their lessons to accommodate different learning needs of learners (T4, Appendix 18).

5.8.3 Improved the attainment of children

The data analysis of the questionnaires indicated that although 85% of the teachers either strongly agreed or agreed, 15% either disagreed or strongly disagreed with the use of ICT to improve the attainment of children Furthermore, in the interviews, the teachers indicated that integrated ICT lessons improved the attainment of children (T5, Appendix 19), confirming the views held by Ertmer et al., (2012). However, Wittwer and Senkbeil (2008) cited in Danhui and Luman (2016) argue that ICT has no direct impact on performance. This claim is questionable in light of the teachers' comments. Apart from the higher attainment perceived by teachers, a large body of empirical evidence accepts that teaching is improved when ICT is integrated into lessons (Abbott, 2001). The tension raised in the 'Think Again' Report indicates that the 'increase use of ICT ' will increase the gap between high and low school achievements' in shape contrast to what is already known and speculated by the teachers.

5.8.4 ICT enhance interactivity and exploration

The data analysis of the questionnaires indicated that although 95% of the teachers either strongly agreed or agreed, 5% either disagreed or strongly disagreed with the use of ICT enhances learning through interactivity and exploration practices. An additional benefit of integrating ICT is to enhance interactivity in lessons (Phillips & Scrimshaw, 1997). This view was reinforced by most of the teachers (T2, Appendix 16; T5, Appendix 19). According to T3, lessons taught using ICT promote 'interactivity and instant feedback to class work' (T3, Appendix 17). In addition, most teachers agreed with Gardner's (1993) views on using ICT to promote multiple intelligences.

5.9 Research question 2: What are the barriers to ICT in mathematics educations?

This session examines the data generated through both the questionnaires and the semi-structured interviews on the views of teachers on the barriers of using ICT in mathematics education. The barriers to ICT integration include the lack of curriculum resources, hardware problems, and teachers' negative attitudes due to lack of experience and support.

5.9.1 Lack of curriculum resources

The data analysis of the questionnaires indicated that although 86% of the teachers either strongly agreed or agreed, 14% either disagreed or strongly disagreed that the lack of curriculum resources was a barriers to integrating ICT pedagogy in mathematics educations. In addition, T3 (Appendix 17) indicated that the absence of ICT curriculum resources is an obstacle to using ICT, as pointed out by Abbott (2001) in the literature review. The need for teachers to find suitable resources toiled to the aims of lessons makes it impossible to frequently integrate ICT pedagogy. School teachers 'not having the skills to develop the right resources and the lack of skills to use them were identified as an obstacle to ICT integration (T5, Appendix 19).

5.9.2 Software and hardware problems

The data analysis of the questionnaires indicated that 100% of the teachers either strongly agreed or agreed that the lack of software and hardware were a barrier to integrating ICT pedagogy in mathematics educations. Commenting on this source of frustration, Johnston-Wilder (2005) points out that although training workshops increase teachers' confidence in ICT integration, most teachers who are themselves class teachers complained about the lack of implementation due to software and hardware problems. T2 (Appendix 16), supported by Graham (1993), indicated that the cost and reliable ICT tools are further problems, including issues related to both out of date software and obsolete hardware problems, poor internet connections and poor interactive white boards continue to discourage teachers from using ICT pedagogy. Added to this, teachers cited that schools must improve upon the ICT infrastructure through the acquisition of up to date hardware and software to resolve these problems.

5.9.3 Lack of experience with ICT tools

The data analysis of the questionnaires indicated that although 85% of the teachers either strongly agreed or agreed, 15% either disagreed or strongly disagreed that the lack of experience with the use of ICT tools was a barrier to integrating ICT pedagogy in mathematics educations. A further barrier is the lack of teachers' own expertise on the use of technology (T3, Appendix 17). The lack of the right up to date knowledge to use ICT tools is a frustration perceived by most teachers in the application of ICT-integrated lessons. The inability to find the required ongoing specialist support on using ICT application tools has made most teachers to develop a poor attitude towards the use of ICT in lessons.

Similarly, the review indicated that the lack of teachers, experience and interest are determinant factors to ICT integration. (T4, Appendix 18). It was widely reported that teachers are likely to use ICT when they had prior skills in using ICT devices (T5, Appendix 19). This agrees with the research conducted by Pelgrum (1993), which revealed that school teachers who are in favour of ICT integration out of school were enthuse to incorporate ICT in their pedagogy.

5.9.4 Negative attitudes

The data analysis of the questionnaires indicated that 100% of the teachers either strongly agreed or agreed that the teacher's negative attitudes were a barrier to integrating ICT pedagogy in mathematics educations. The implication for teachers is that they lack the attitude and the desire to promote ICT-integration (Mingaine, 2013). Most teachers agreed with the view that, time should be set aside for training staff with a clear policy on ICT integration.

5.10 Research question 3: What are the facilitators to ICT into mathematics education?

This session examines the data generated through both the questionnaires and the semi-structured interviews on the views of teachers on the facilitators of using of ICT in mathematics education.

5.10.1 The availability of ICT tools

The data analysis of the questionnaires indicated that although 95% of the teachers either strongly agreed or agreed, 5% either disagreed or strongly disagreed that the up to date provision of ICT tools will facilitate the integrating of ICT pedagogy in mathematics educations. In answering the above question, it was interesting to discover that most teachers frequently mentioned in the interview that the provision of visual and dynamic aids, including videos and films, interactive whiteboards, YouTube videos, and tablets as the main facilitators of integrating ICT into mathematics pedagogy (T1, Appendix 15).

5.10.2 Experience and knowledge of using ICT

The data analysis of the questionnaires indicated that although 95% of the teachers either strongly agreed or agreed, 5% either disagreed or strongly disagreed that the experience and persona knowledge of using ICT will facilitate the use of ICT pedagogy in mathematics educations. **Also,** the interview questions confirmed the perception that teachers own experience and personal knowledge of using ICT were the main facilitators of using ICT (T2, Appendix 16; T4, Appendix 18), a finding consistent with the literature review (Ellis & Loveless, 2001).

5.10.3 School leadership

The data analysis of the questionnaires indicated that although 90% of the teachers either strongly agreed or agreed, 10% either disagreed or strongly disagreed that the school leadership on ICT will facilitate the integrating of ICT pedagogy in mathematics educations. The teachers' responses from the interview also confirmed that the attitudes of school leaders play a major role in determining the extent of ICT involvement in their lessons (Fullan et al., 1988). The participants acknowledged the need for school leaders to have a clear ICT policy initiates in lessons and designated staff to induct teachers who lack the required ICT expertise (T5, Appendix 16). Many teachers commented on the existing school leaders' styles. This corresponds with the views of O'Reilly and Reed (2010), who claim that the application of ICT policies depends on the style of school leaders. Again Fay was of the view that a leader that adapts the transformational style in the implementation of any school policy will achieve good results (Fay, 1987). In addition, a leader who operates under the collegial approach will be able to influence teachers to adapt a school policy with confidence (Bush, 2011). Generally, most teachers stated the need for school leaders to offer teachers plenty of chances to experiment with new technologies and provide ongoing support to reflect expertise (Fullan, 2006) before any classroom implementation.

5.10.4 Opportunities for training

The data analysis of the questionnaires indicated that although 94% of the teachers either strongly agreed or agreed, 4% either disagreed or strongly disagreed that the opportunities for training will facilitate the integrating of ICT pedagogy in mathematics educations. School leaders must provide opportunities, incentives and workshops to motive the integration of ICT. The training can be provided either on-the-job or off-the-job. The benefit of training for teachers is that it can then change the attitudes of teachers (Dawson & Rakes, 2003). However, surprisingly, most teachers reported that the lack of appropriate professional development has resulted in a lack of confidence in the ICT skills provided whilst training as a teacher. This corresponds with the literature review finding that most teaching agencies offered typically too short courses with no follow-up.

5.11 Chapter Summary

The above evaluation on relationship between this research, questionnaires, case studies and the semi structured interview indicates that the findings of this study are fully supported by the findings of previous studies. Having no contradictions to previous research demonstrates that the findings of the study are not biased, but genuine. Therefore, from the discussion and evaluation of the findings, it can be stated that the impact of ICT in mathematics education is dependents on a number of factors, including the benefits, barriers, support, and facilitators of ICT. Other factors include the style of teachers and the available professional support to equip teachers with the required confidence in the development of ICT pedagogy. Teachers need a clear vision with ongoing training to improve their attitudes towards integrating ICT into mathematics pedagogy.

CHAPTER 6: CONCLUSION

6.1 Introduction

The aim of this study was to explore the question: 'What are the impacts on ICT in mathematics education? In order to understand these impacts, the study conducted a review of the existing literature in light of the available information obtained in interviews through primary research. To respond to the research question, the study examined two case studies on own practice, questionnaires and interviews of teachers. The following sub questions guided the entire research;

- What are the benefits of ICT in Mathematics education?
- What are the barriers to ICT in mathematics education?
- What factors will facilitate ICT in Mathematics education?

The deductive research approach and the descriptive design of the research enabled detailed content analysis and investigation of the research questions. The research used both the primary data as well as the secondary data set through a mixed methodology approach. In the literature review, the study identified that previous research has indicated several benefits that school teachers gained if ICT was included within their mathematics pedagogy. Some of the benefits included easy problem solving and interactive lessons. At the same time, the literature review indicated some barriers that may exist while including ICT in mathematics pedagogy. Some of the barriers included lack of ICT knowledge among teachers, lack of ICT hardware, and associated software problems.

6.2 Effect digital tool

An ICT dynamic software tool such as GeoGebra has features that allow quick and easy creation of mathematical objects presented as visual representations. GeoGebra software supports the collaboration and the creativity of mathematical thinking and reasoning. It provides the students with a problem-solving method and a shared working space and feedback of the respective actions on the space.

Students reported that they found the use of ICT tool helpful in testing ideas and concepts by formulating input and submitting according to a question hypothesis. In the lessons examined in this study, it is clear that as the learners interacted with GeoGebra, they benefited with the understanding of the topics better. There is, however, a factor that affects the effectiveness of employing ICT in lessons. It is, yet, vital to realise that when students utilised ICT to work on their tasks they participated in creative reasoning and therefore were able to demonstrate a higher understanding in mathematics

6.3 TPACK Framework

While many researchers find in the TPACK model a worldwide model for prospectus design, I found it awkward, confusing and misleading to sort out into clear groups of TK, PK, CK, TPK, PCK, TCK and TPACK, to enable me advance with the lesson design. Mention can be made of the empirical study comprising 596 online teachers in the United States whose views were studied on the practicality of using TPACK (Archambault & Barnett, 2010). According to this report, just as I found it in my lesson design, it was difficult to verify knowledge into pedagogy, content and technology as three separate domains. The only obvious classification of knowledge was the domain that segregates itself from the others, technology.

I agree with the view that the isolation of knowledge into the TPACK framework is not obvious and not a theory that is possible to empirically verify. This is because throughout the review it was identified that TPACK has no reliant or autonomous defined variables since all available constituents are classified as identical with no clear conditional or causal inferred associations. The non-existence of any null-hypotheses to examine with no known evidence-based results to predict makes the concept even more confusing to justify.

TPACK was unsuccessful when used in many applied researches in the initial empirical research when knowledge was to be distinguished into the seven equally detached domains. Even most worryingly, however, is the inability of the model to propose either an aim-directed structure that is linked with the authorisation of learners in the usage of technology or the relationship around the impact of digital liberalisation routes on society. It is worth mentioning that TPACK needs to be established so that the optimistic impact credited to the intrinsic effects of the method not founded on the critical thoughts are certain to attain when teachers deliberate on the incorporation of dynamic technologies into their pedagogies.

Using the strengths of the Knowledge Quartet to complement TPACK was eye-opening. This framework assumes the view that ownership of knowledge enlightens pedagogical strategies and choices in a fundamental way with the key components being knowledge and understanding of mathematics. The Knowledge Quartet offered a framework to review and analyse my lessons from a mathematics point of view. Using the framework assisted me to concentrate on my mathematics contents instead of the technology, which was accommodated through the TPACK framework.

6.4 Lesson learned

The lessons learnt through the use of the framework have paved the way for me to understand the needs of my learners better when ICT is used and has equipped me to plan sequences of lessons aimed at engaging my students. Using the Knowledge Quartet granted me the needed objective view on the plight of students' mathematical understanding when ICT software is integrated in the learning process. In this manner, my technique towards the pedagogy of the subject becomes modified after each practice, hence epitomising the ideas of Scott et al., (2003), who pointed out that such a practice is able to promote a conjectural loop existing between philosophy and practice. The insights gained on my own practice were valuable and I hope will be used as a catalyst to improve my mathematics subject knowledge.

6.5 Summary of findings

While discussing the relevance of ICT in mathematics pedagogy from the professional point of view, it was vital to use the responses of teachers and students to clearly understand that insights of ICT integration were important for teacher implementation, as opinions influence attitudes, and attitudes affect behavioural intentions (Polizzi, 2011). An ICT integrating learning in mathematics education is 'successful if it is accompanied by social and institutional change in educational settings' (Punie et al. 2006:37). Even though to support ICT practice in mathematics, institutional policies should be redefined to support ICT pedagogy, policy makers must also 'acknowledge that the impact of ICT is complex and uncertain' (wood, 2003; 2).

The study has revealed the planning of ICT lessons in mathematics must appreciate the importance of prior knowledge on ICT before any implementation. The difficulties held by most teachers could be resolved through ongoing developments to address these challenges. Notwithstanding, the number of conclusions that can be drawn concerning teachers' attitude towards the impact of ICT integration into mathematics pedagogy, my initial stance was that the confidence of teachers influences the level of ICT usage in the lesson. Admittedly, after carefully analysing the facts, I am of the view that other factors including the initial impact on ICT use is contradictory in light of the empirical evidence backed by the primary data. The review revealed that teachers' perceptions on the impact are independent of their attitudes towards ICT integration. This refutes the opinions of most teachers interviewed, and I believe that there is a relationship between teachers' attitudes and the implementation of ICT use in lessons. Institutionalised resistance also affects 'subliminally and therefore without acknowledged intentionality' happening consciousness impacts on ICT use in schools (Somekh, 2008:116). However, given the appropriate provision of relevant technologies, attitudes and behaviour can change over time (UNESCO, 2003).

6.6 Limitation and difficulties

No doubt combining a mixed methodology approached in this study was very cumbersome to do. I must knowledge that all of the information gained from such a wide range of literature including questionnaires and interview responses cannot possibly be presented in a single research paper due to time constraints and resources. Many problems were encountered in embarking on this study. The greatest was the difficulty of securing appointments to conduct interviews with teachers. I found it very frustrating with the numerous interview appointment cancellations that I had to go through before being able to conduct this research. Added to this was the huge amount of time required to transcribe the interviews from the tapes. Also I am uncertain that I was able to capture every detail as communicated by the participants.

Notwithstanding, designing the lesson was compounded with many technological problems. Among the problems were the lack of up to date computers and the latest version of the GeoGebra software. This problem created difficulties in the delivery of the lesson through loss of teaching time. Added to this was the unavailability of qualified staff to support students when faced with software difficulties. I believe that when these and many barriers of ICT lessons already mentioned are resolved teachers will be motivated to integrate technology in the teaching of mathematics.

6.7 Recommendations for further research

Finally, I have found this experience to be worthwhile, and the research experience has made me appreciate empirical inquiry. It has also taught me valuable critical thinking skills and enabled me to meet the skills required by the job market. As a teacher, I felt honoured to research and contribute to a topic that is known to have caused controversies within mathematical educational. This satisfaction motivates me to continue conducting research into interesting topics in whole school education such as “ICT across the whole school”. Through this, I will gain the needed confident to lead the whole school initiative on ICT learning.

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Appendices

Appendix 1: Case study 1

Applying Technological, Pedagogical Content Knowledge to Lesson

TPACK framework is based on three primary forms of knowledge. So my first step was to understand the primary forms of knowledge in the context of the lesson.

- Content Knowledge (CK)—what am I teaching and what is my own knowledge of the subject? For this lesson, I needed to establish a solid understanding of circle theorem and processes.

- Pedagogical Knowledge (PK)—how do my students learn best and what instructional strategies do I need to meet and what are the requirements of the lesson plan? In this case, I needed to understand the best practices for teaching secondary school year 10 mathematics using small group collaboration and the preferred learning styles in my class.

- Technological Knowledge (TK)—what digital tools are available to me, which do I know well enough to use, and which would be the most appropriate for the lesson at hand? Geogebra. For this lesson, students will explore through investigating the proofs associated with circle theorems. The ability to calculate angles and justify the reasons for answers are important.

By looking at the primary knowledge and focusing on the intersection whilst viewing the goal of the lesson and the strategy through the lens of TPACK, I took a moment to consider the individual relationships.

- Pedagogical Content Knowledge (PCK)—understanding the best practices for teaching specific content to specific students in my class.

- Technological Content Knowledge (TCK)—knowing how the digital tools available to me can enhance or transform the content, how it’s delivered to students, and how my students can interact with it.

- Technological Pedagogical Knowledge (TPK)—I had to think about using the digital tool as a vehicle to the learning outcomes and implement the desired experiences.

- Finally, the technological, pedagogical content knowledge (TPACK) was put together to enhance the activities of the lesson plan.

Below are examples of activities that were considered;

- After walking through the different parts of a circle theorem,
- Put students into small groups of three and asked them to collaborate on completing a Check for Understanding quiz via your GeoGebra; an interactive question that provides a diagram of a circle with blank labels and requires students to drag and drop the proper labels in place from an answer key.
- Students will compare the different circle theorem properties and make conclusions regarding the differences they find.
- Students will be asked to analyse the connections between different circle theorem. Each group will be asked to infer what might happen when different circle properties are considered. Each group will compile evidence to make their case.

Appendix 2: Activity Sequence:

Lesson 1

Learning objective: To learn by investigations;

- Task 1: The angle at the centre of a circle is twice the angle at the circumference of the circle from the same arc.
- Task 2: The angle formed in a semicircle is always a right angle. (Task 2)
- Task 3: Angles from the same arc in the same segment are equal. (Task 3)

Whole-class starter activity: Project Geogebra on the IWB to emphasize on the general functions of the Geogebra – (Students to work through starter activities in pairs) -15 minutes

Class work: Task 1, Task 2 and Task 3 - 30 minutes

Feedback: Whole-class discussions on completed activities with misconceptions resolutions – 15 minutes

Extension: - Task 4: Opposite angles of a cyclic quadrilateral add up to 180^{O}.

Strategies and activities:

Teacher: Walk through the circle theorems and the basic properties of circle, noting the diagram in pupil's exercise book

Activities: Break the class into small groups. Task each group with labelling their own diagram of circle and researching a single theorem to present to the class later on.

Ask each group to present the circle theorem on the IWB to the class.

Lesson 2

Learning objective: To learn by investigations;

- Opposite angles of a cyclic quadrilateral add up to 180^{O}. (Task 4)
- Two tangents drawn from a point to a circle are equal. (Task 5)
- The angle between a tangent and a chord is equal to the angle at the circumference in the alternate segment. (Task 6)

Starter activity:

Project Geogebra on the IWB to consolidate the key operations of the Geogebra functions -15 mins

Students to work through starter activities in pairs – 15 minutes

Whole class work: 30 minutes (**Task 4, Task 5, Task 6)**

Extension: - Work sheet: To investigate; (The perpendicular line from the centre of a circle, chord and the bisection to the chord. The angle between a tangent and the radius, at the point where the tangent touches the circle. (Is it a right angle?)

Feedback: Whole-class discussions on the completed activities Major activities

Teacher: Walk through the circle theorems and the basic properties of circle, noting the diagram in the textbook

Activities: Break the class into small groups. Task each group with labelling their own diagram of circle and researching on the circle theorems to present to the class later on

Appendix 3: Task: Circle Theorem

Task 1: The angle at the centre of a circle is twice the angle at the circumference of the circle from the same arc.

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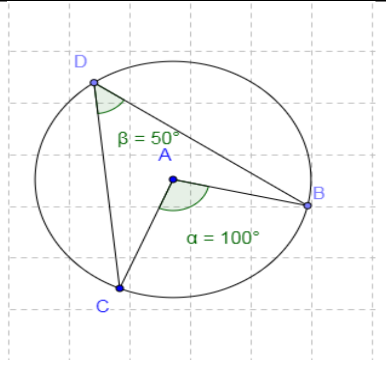
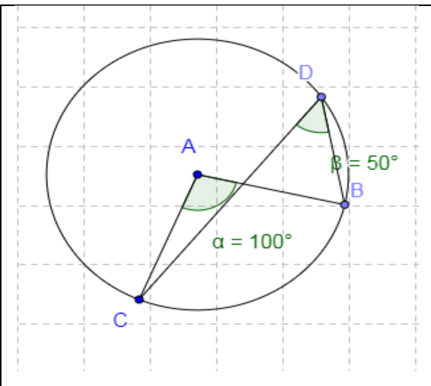
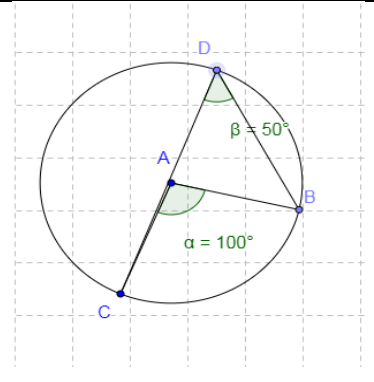
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Figure 1a</td>
<td>Figure1 b</td>
<td>Figure1 c</td>
</tr>
<tr>
<td colspan="3">What's the connection between the angle at the centre, α, and the angle at the circumference, β?
You can use the cursor to move point B, and hence to alter angle α.
You can also move point D around the circumference. If you move point C, then angle α and the size of the circle both change. Notice what happens to angle β as you make these changes.</td>
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Figure 1: Visualizing the inscribed angle theorem.

Task 2: Angles inscribed in a semicircle are right angles

<table>
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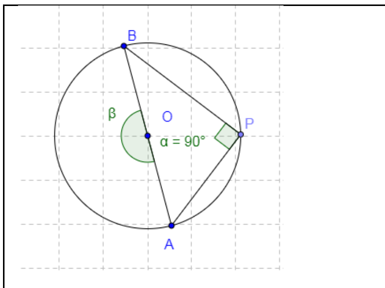
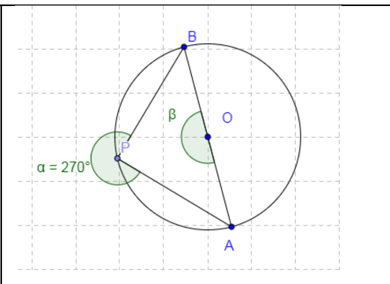
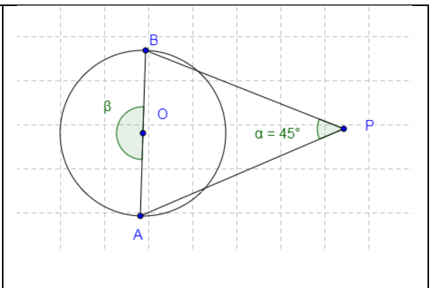
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Figure 2a</td>
<td>Figure 2b</td>
<td>Figure 2c</td>
</tr>
<tr>
<td colspan="3">Figure 2b: What happens to angle α as you move P? (An angle bigger than acute, but smaller than obtuse).
Figure 2c: For the first one, try moving the point P about. Where does P have to be for angle α to be acute? and where must P be for angle α to be obtuse? So where must P be to make angle α = 90 degrees?</td>
</tr>
</table>

Figure 2. Visualizing the Angles inscribed in a semicircle are right angles theorem

Task 3: Angles in the same segment are equal.

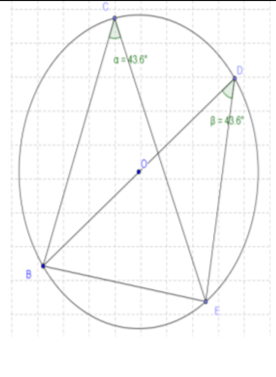


Figure 3a

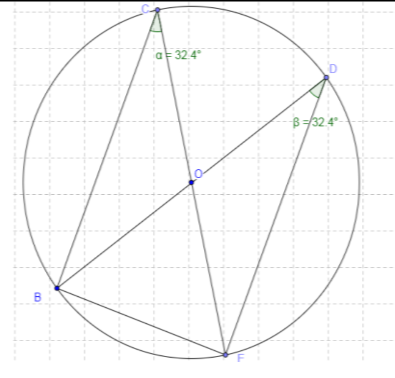


Figure 3b

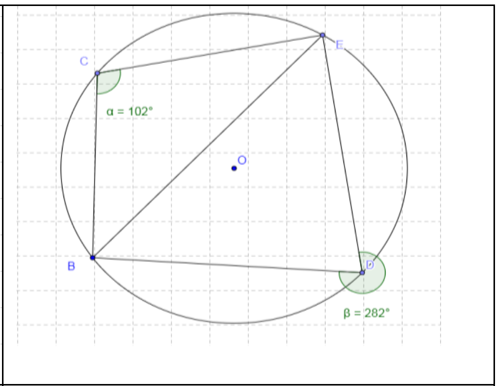


Figure 3c

Compare the sizes of angles α & β, as you use the cursor to move point C or D around the circumference.

It doesn't change either angle β- they stay the same!

What is the effect of moving point E?

And finally, what happens when you move point B?

Figure 3: Visualizing the Angles in the same segment are equal.

Task 4: Cyclic Quadrilateral Theorem

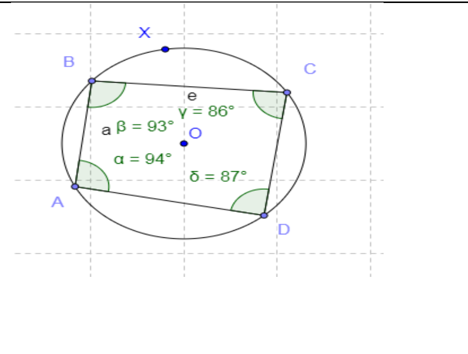


Figure 4a

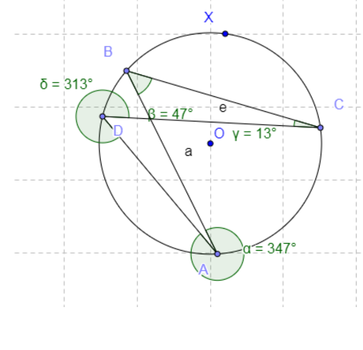


Figure 4b

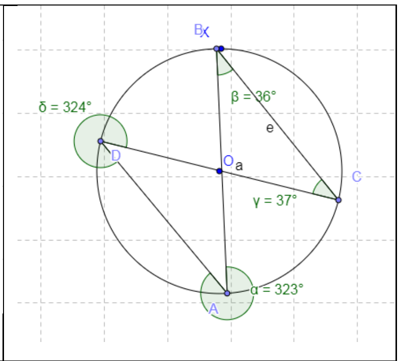
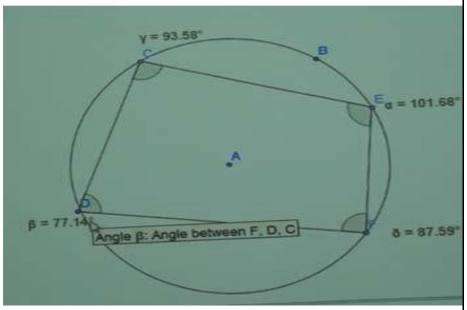


Figure 4c



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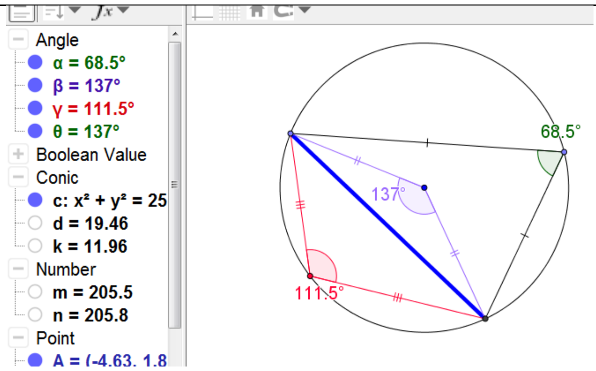
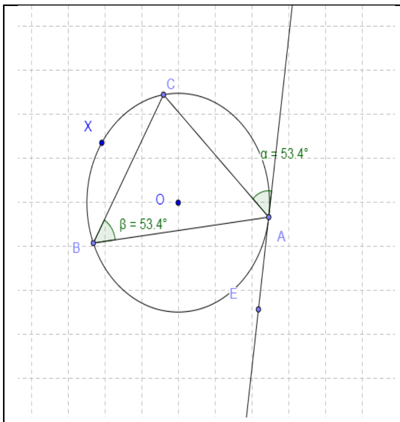
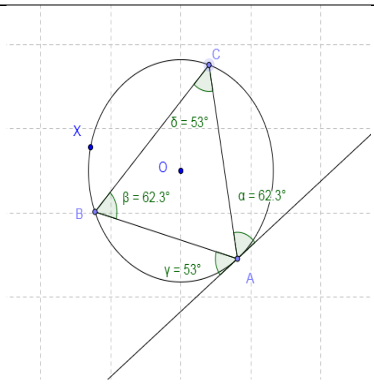
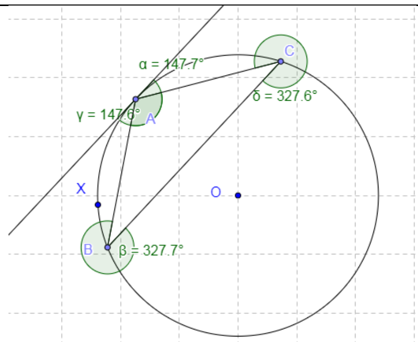
<tr><td></td><td>
</td></tr>
<tr><td>Figure 4d</td><td>Figure 4e</td></tr>
<tr><td colspan="2">Figure 4a : What's the connection between pairs of opposite angles (α and γ or β and δ)? ***
You can use the cursor to move any of points A, B, C or D around the circumference, and hence alter some of the angles.
You can also move point X or O to change the size of the circle.
Figure 4b
Figure 4c: Agreeing with task 3, angles in the same segment are the same</td></tr>
</table>

Figure 4: Visualizing the Cyclic Quadrilateral Theorem

Task 5: Alternate Segment Theorem

<table>
<tr><td>
</td><td>
</td><td>
</td></tr>
<tr><td>Figure a</td><td>Figure b</td><td>Figure c</td></tr>
<tr><td colspan="3">You can use the cursor to move the point B around the circumference, but it doesn't change things much, does it?
So you can try moving point C around the circumference, and hence alter the angles.
The connection between angles α and β should be clear!</td></tr>
</table>

Moving point A changes the position of the tangents.

Moving points O or X changes the size of the circle.

Figure 5: Visualizing the Alternate Segment Theorem

Extension work:

<u>Task 6. Perpendicular & chord</u>

A perpendicular line from the centre, O, cuts the chord CD of the circle at point B .

So OB is perpendicular to the chord CD.

Move C or D, and note the connection between lengths CB and DB.

If you want to alter the size of the circle, move O or X.

<u>Task 7. Angle between tangent and radius</u>

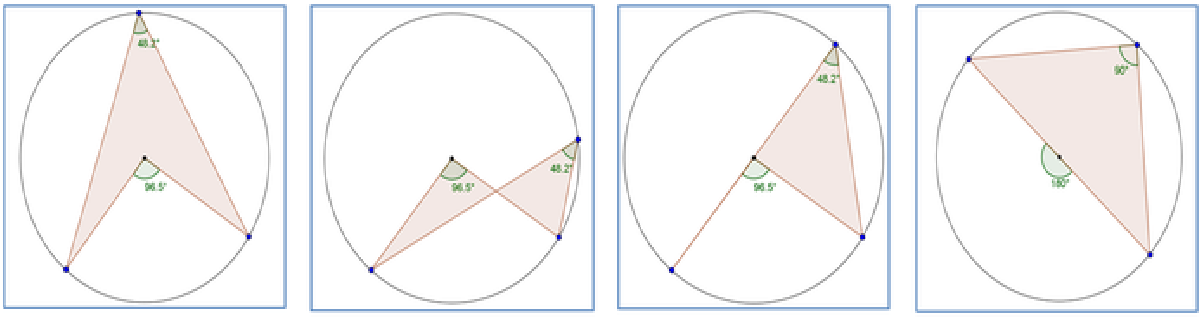
What do you notice about the angle between tangent and radius, α & β? You can use the cursor to move point C, and you can change the size of the circle by moving point O or X.

<u>Misconceptions:</u>

<u>Adapted from: (https://danpearcymaths.wordpress.com/2019/09/20/flipped-classroom-circle-theorems-and-common-misconceptions/)</u>

The angle at the circumference is half the angle at the centre.

4 separate cases to consider understanding the misconceptions

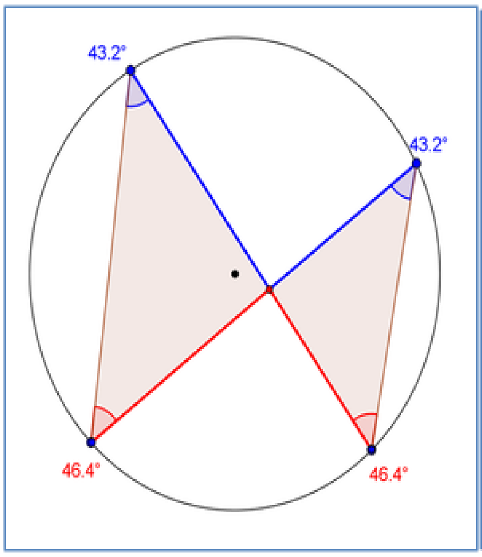


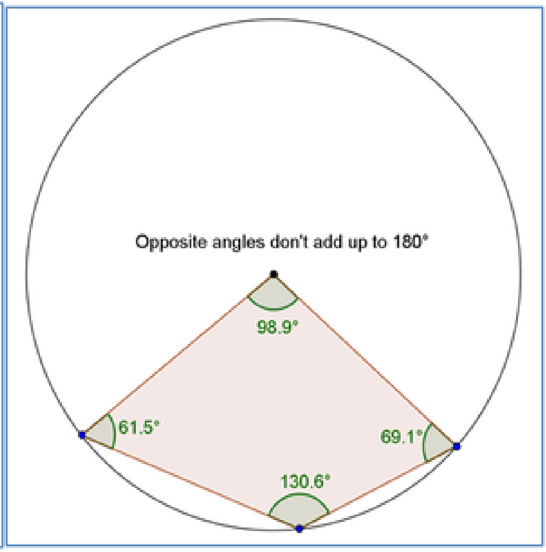
Case 1 Case 2 Case 3 Case 4

Case 1: Often presented in the textbooks and easier to understand by students

Case 2 Angles on the circumference sharing the same arc are equal. Most students cannot identify the two similar triangles when they associate the shape to the bow-tie angle theorem as seem below. Here the triangles are not similar.

Also it is difficult for students to tell which angles are equal. Because they fail to realise that when the triangles are similar the angles which lie on the same **arc** will be equal as seen below.





What are the misconceptions with the cyclic quadrilateral theorem?

Students make a mistake because the opposite angles in the quadrilateral does not always have to add up to 180 degrees. It is only possible when all the vertices of quadrilaterals lie on the circumference of the circle.

Appendix 4: Case study 2

Lesson plan 1

Introduction

Title: Demonstration of drawing lines

Recape: Review the aim of the lesson Awareness of purpose; Use GeoGebra to draw basic Lines and find the Equation of lines- Time 15minutes

Demonstrate the misconception in drawing lines and graphs: identifying errors; overt subject knowledge; theoretical underpinning of pedagogy; use of terminology; Class textbook to understand example on functions: use of textbook;

Class work: Practical Exercise: Controlling Characters with Graphs: reliance on procedures

Corner stone Activity 1: Coordinating algebraic, graphical, and tabular representations

Making connections between procedures;

making connections between concepts; anticipation of complexity; decisions about sequencing; recognition about conceptual appropriateness.

Speed as a context to introduce rates of change y= mx+c as a model of constant velocity motion – the meaning of m and c in the motion context

Choice of representations and explanations;

choice of examples, teacher demonstrations

Lesson plan 2

Review of previous lesson

Discuss Key words: use of terminology;

Lesson objectives: Graphing Linear Functions and finding the relations between tables and algebraic representations. Awareness of purpose;

identifying errors; overt subject knowledge; theoretical underpinning of pedagogy;

Demonstration: Creating a y-y1=m(x-x1) graph on GeoGebra (15minutes)

Completing examples from text book: use of textbook; reliance on procedures

Practice 2

Graphing Linear Functions with GeoGebra (10 mins)

- Awareness of purpose;

Activity2: Money remaining and the days left for the holiday

choice of examples, teacher demonstrations

Responding to children's ideas; deviation from agenda;

swift and correct analysis of student errors and difficulties

Creating a linear sequence in GeoGebra (using spreadsheets)—10 minutes

Demonstrate the relations between functions, tables and symbolic representations.

Choice of representations and explanations;

choice of examples, teacher demonstrations

use of opportunities;

Appendix 5: Coding using TPACK

Section 1: Teacher Lesson Plans for TPACK

The following themes have emerged:

Linear functions Overview of unit

<table>
<tr><td colspan="2">
<u>Key to Knowledge Quartet codes</u>

Foundation, Fo
AtT – adheres to textbook
AP – awareness of purpose
CoP – concentrates on procedures
IE – identifying errors
OSK – overt subject knowledge
TUP – theoretical underpinning
UT – use of terminology

Transformation, Tr
CUE – choice and use of examples
CUR – choice and use of representation
TD – teacher demonstration
UIM – use of instructional materials

Connection, Co
AC – anticipation of complexity
DS – decisions about sequencing
MCC – making connections between concepts
MCP – making connections between procedures
RCA – recognising conceptual appropriateness

Contingency, Cy
DA – deviation from agenda
RCI – responding to children's ideas
UO – use of opportunities
TI – teacher insight
RAT – responding to (un)availability of tools and resources
</td></tr>
<tr><td>
Practice 2
Amount of money spent on vacation
Writing an equation
Money remaining and the days left for the holiday can be
represented as a straight line
View setting Drag and drop the graph
Set specific windows
</td><td></td></tr>
</table>

Using function equations Improve the view using different colures Finding the interceptions of lines Click on points and the segment Label axis Use the manual box to label axis Identifying the money remaining by labelling the axis Using the export function in GeoGebra Using copy and paste functions Controlling Characters with Graphs Graphs are mathematical representations of relationships such as savings Play and pause a simulation. For graphs of savings (that is, money versus days), the steeper the line, the lesser the money savings. Demonstrate to students how to modify the graph to change the savings. The amount of savings can be determined from different parts of a graph and model. Adjust the graph to change the final position. Representing the Equations in the form of Graphs and tables. Play and pause a simulation. Writing equations based on tables or graphs. Modify the graph to change the savings. Translate between tables, graphs and equations. Demonstrate how to adjust the graph to change the final position. Representing days, money, holidays and savings differently in three demonstrations. Play and pause a simulation.	Fo/AP- awareness of purpose Fo/UT-use of technology Tr/TD- teacher demonstration Tr/CUR-use of representation Co/MCC- making connections between concepts Cy/TI- teachers insight

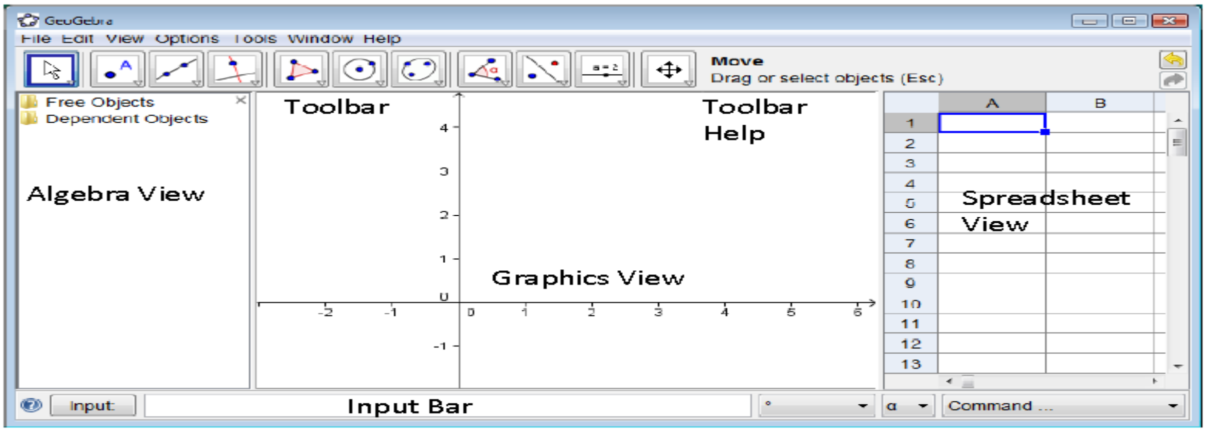
For equations of the form y = mx + c, Saving y and money and days respectively Modify the graph to change the savings. Demonstrate how to translate between graphs, tables and algebraic expressions. Adjust the graph to change the final position.	

Section 2: lesson plan with Knowledge of Quartet

Foundation	Awareness of purpose; identifying errors; overt subject knowledge; theoretical underpinning of pedagogy; use of terminology; use of textbook; reliance on procedures.
Transformation	Choice of representations and explanations; choice of examples, teacher demonstrations
Connection	Making connections between procedures; making connections between concepts; anticipation of complexity; decisions about sequencing; recognition about conceptual appropriateness.
Contingency	Responding to children's ideas; use of opportunities; deviation from agenda; swift and correct analysis of student errors and difficulties

Appendix 6: Snapshot from lessons

INTRODUCTION TO GRAPH DRAWING WITH GEOGEBRA



Drawing graphs

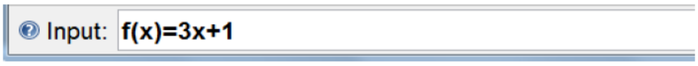
The procedure is similar for all types of functions. First write the desired function expression in the input bar. Then, change the axes so that the graph show up in the graphics view.

We will now draw the graphs of the to following functions:

1. The straight line given by $f(x)=3x+1$

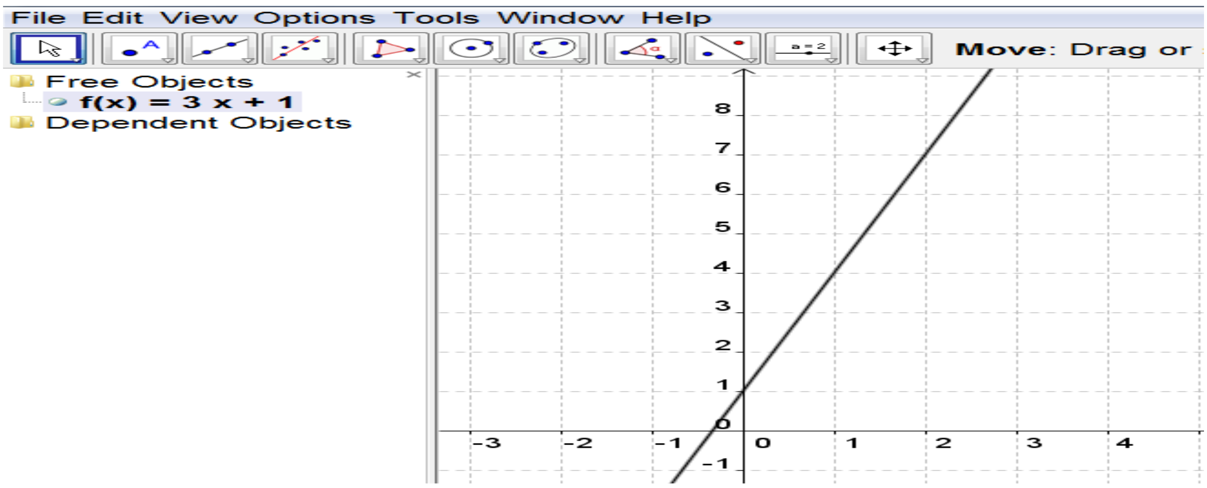
1. The straight line $f(x)=3x+1$

In the input bar, type f(x)=3x+1.



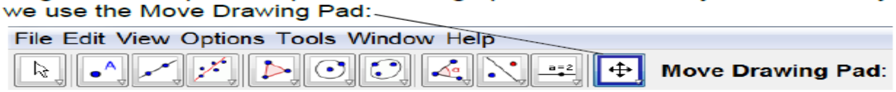
f(*x*) is the name of the function *f* of the independent *x*. We can of course provide other names for the function like *g*(*x*), *h*(*x*), *k*(*x*). Note that we do not need the multiplication sign between 3 and x.

Press enter and the graph of the function will appear:

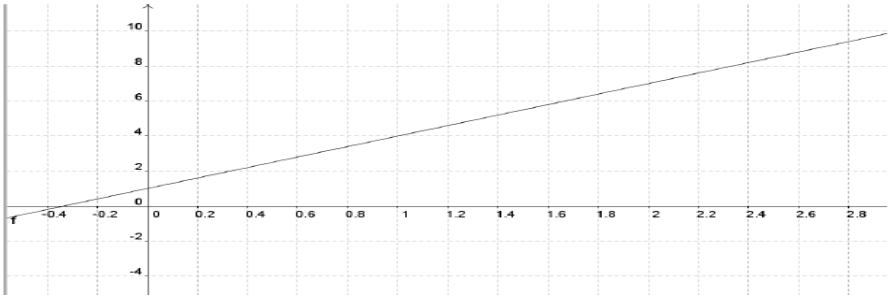


Adjusting the x- and y-axis:

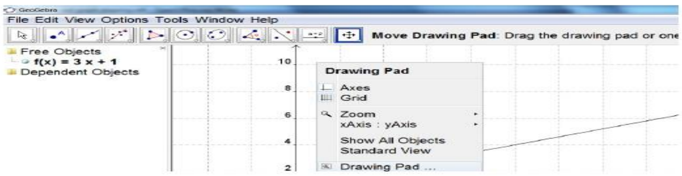
To get the best possible picture of the graph, we need to adjust the x- and y-axis. For this we use the Move Drawing Pad:



Press this button so that it is highlighted. Move the cursor so that it lies directly over one of the axes. Press left mouse button and drag the axis to change the units. Try to make the drawing pad look like this:

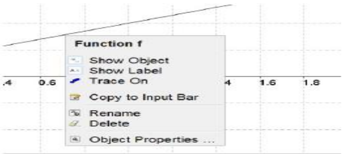


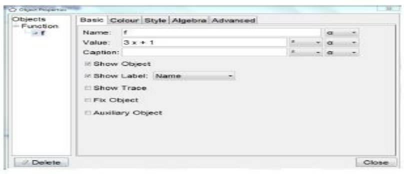
Editing in the graphic view



Select Drawing Pad to adjust axes, grid etc. Try i out!

Right click on an object in the graphic view and choose Object Properties for more editing choises on already drawn objects:





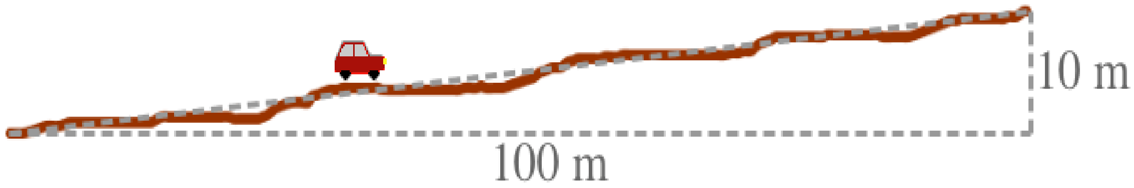
Linear Functions- Lesson and Exercise

Aims: 1. To understand the concept of Slope

2. Calculate gradients and comment on the nature of slope

Starter lesson:

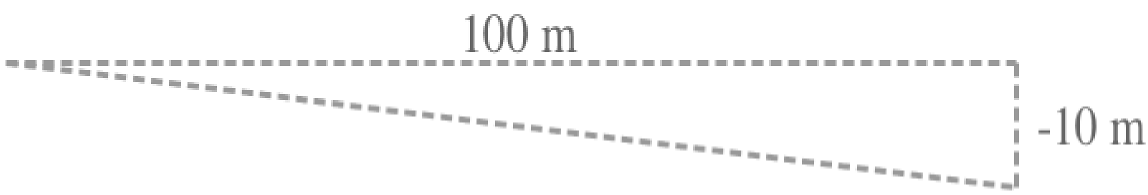
The ten percent slope in the road signs above mean that if you move 100 m along the horizontal direction, then you move at an average 10 m along the vertical direction.



In order to find the percentage slope, you divide the movement along the *y*-axis with the movement along the *x*-axis, 10100=0.1=10%.

The road signs above assume that you drive from the left to the right, from the look of the pictures you can deduce whether it is a slope downwards or upwards.

When defining slope in mathematics, the definition is similar to the slopes on the road signs. The slope is defined to be the difference along the *y*-axis divided by the difference along the *x*-axis. In order to distinguish between a slope downwards and a slope upwards, the difference along the *y*-axis is counted as negative if the slope is downwards when going from the left to the right.



Instead of the words "difference along", the symbol Δ is used. The difference along the *y*-axis is denoted by Δy and the difference along the *x*-axis by Δx. If m denotes the slope, we get following formula: m=Δy/Δx

Exercises

Exercise 1 - Slope by definition

Start a new GeoGebra sheet. Right-click on the drawing pad and check both Axes and Grid.

Choose Options->Point Capturing->On (Grid).

Use the tool **Line through Two Points** to make four lines through the points:

1. (-1,1) and (3,4)
2. (0,-1) and (4,-1)
3. (2,-2) and (-2,2)
4. (1,1) and (1,4)

Find the slope of the four lines by using the definition $m=\Delta y\Delta x$ and write down your answers!

Exercise 2 - Using the tool Slope

Use the tool **Slope** on the four lines. Click on the tool and then on a line.

Compare the GeoGebra-slopes to your answers.

Comment

The GeoGebra-way of defining slope could be seen as an alternative definition:

If the change along the x-axes (moving from left to right) is 1,
then the change along the y-axes is the slope.

[Load GeoGebra worksheet]

Slope

Exercise 3

Start a new GeoGebra sheet. Make sure that the input bar is shown; if not, pick View->Input Bar.

Enter following three functions in the input bar (one at the time):

- y=x
- y=x+2
- y=x−1

Without plotting the graph, answer these questions about the function y=x+4.

1. What is the slope?
2. What is the *y*-intercept?
3. What is the *x*-intercept?

Exercise 4

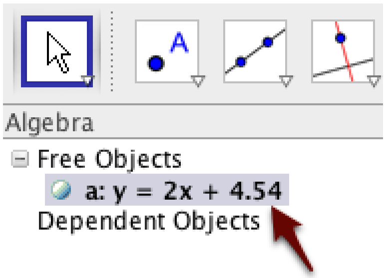
Delete the three functions and instead enter following functions in the input bar:

- y=−2x
- y=−2x+3
- y=−2x−2

Without plotting the graph, answer these questions about the function y=−2x+1.

1. What is the slope?
2. What is the *y*-intercept?
3. What is the *x*-intercept?

Exercise 5



Delete the three functions and enter the equation y=2x in the input bar.

The equation is shown in the algebra view and the line is shown in the drawing pad.

Use the mouse to drag the line and watch the equation in the algebra view. The second number shown in the expression, changes as you moves the graph. Describe how you can find the second number of the expression by looking at the graph!

Exercise 6

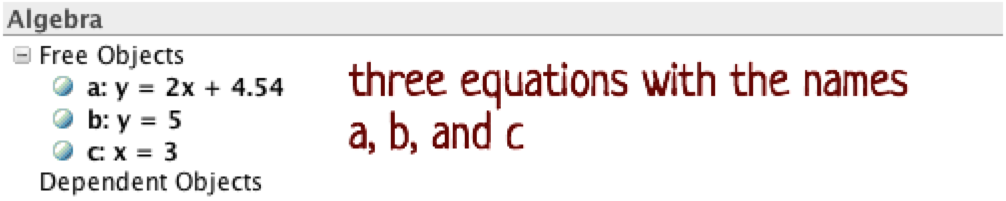
Without drawing a line, answer these questions about the line defined by y=3x−2.

1. What is the slope?
2. What is the *y*-intercept?

Comment

GeoGebra distinguishes between functions and equations. Functions are written like this: f(x),f1(x),g(x),

Main Function (x),... Equations are written using the variables x and y. Each equation is given a unique name, but all equations use the variables x and y.



An equation can be used to define a **line** like this:

Only the points whose coordinates x and y satisfy the equation

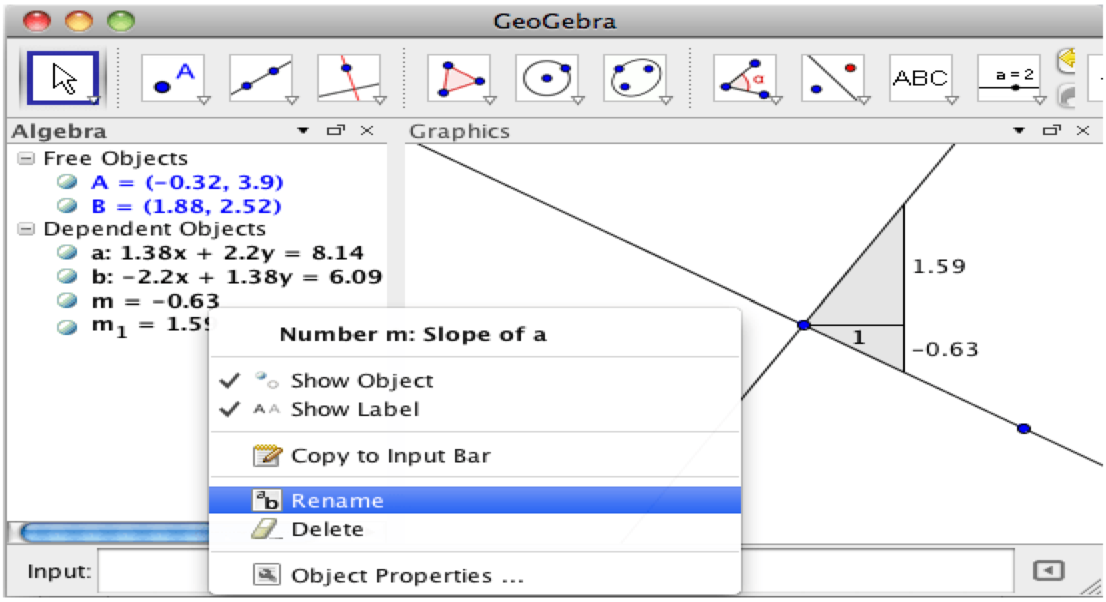
(makes the equality true),

lie on the line

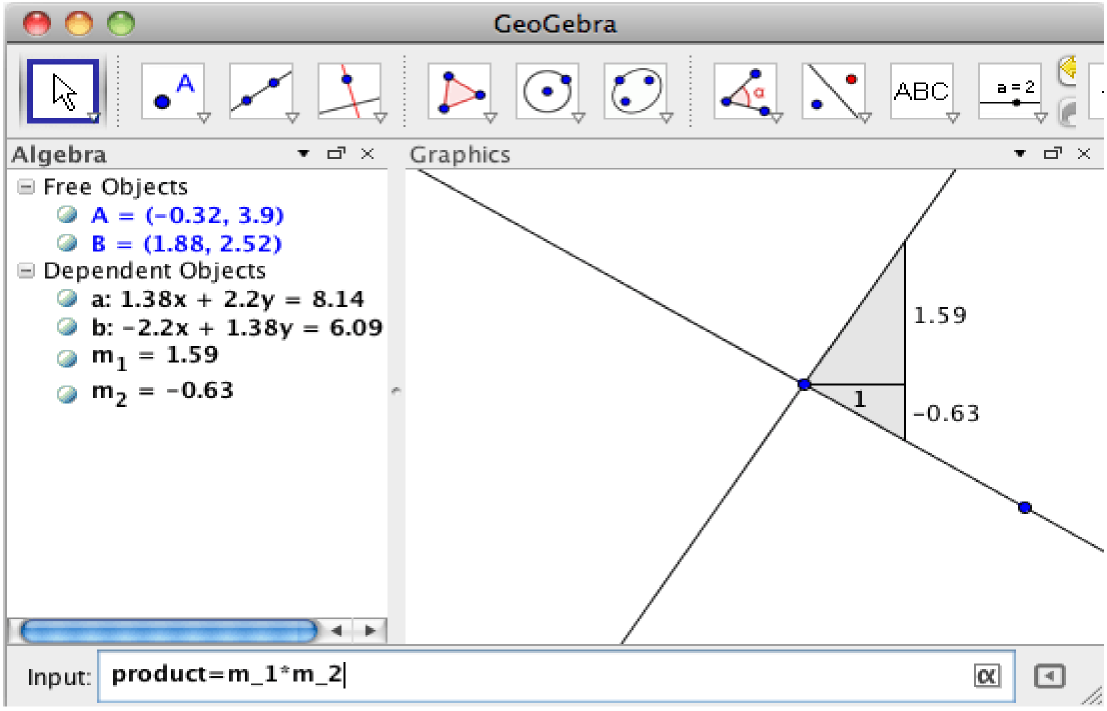
A longer explanation is given on next page.

Gradients of Perpendicular Lines

- Draw a line through points A and B.
- Draw a line perpendicular to the first line through the point A.
- Use the  Slope tool on both lines.
- Rename the gradients to m_1 and m_2 respectively.



- Make a new variable as in the picture below. Move the points!



- Make a conjecture! Prove it!

Hint: Look at the picture and use similar triangle.

reference: the pictures of the road signs are from: http://commons.wikimedia.org/wiki/Category:Slope_signs

Source of lesson: http://www.malinc.se/math/functions/linesasfunctionsen.php

Appendix 7: Interview scripts -students

Knowledge Quartet emphases questions

Question 1:

What do you understand by a function as a rule

Response:

A function assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output

Question 2:

Give me an example of why a function is a rule?

Response:

For example, the rule that takes *x* as input and gives 5*x*+4 as output is a function. Using y to stand for the output we can represent this function with the equation $y = 5x+4$, and the graph of the equation is the graph of the function will be a straight line.

Question 3

What do you understand by linear function?

Response:

Is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

Question 4

Can you give me an example of a function?

Response:

For example, the rule that takes *x* as input and gives 5*x*+4 as output is a function. Using y to stand for the output we can represent this function with the equation $y = 5x+4$, and the graph of the equation is the graph of the function.

Question 5

How would you Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

Response

For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line

Question 6

Can you determine which of the functions listed below are linear and which are not linear and explain your reasoning.

- $y = -2x^2 + 3$ non-linear because it has a square
- $y = 2x$ linear, because it has 2 as gradient and 0 intercept

- $A = \pi r^2$ non-linear, nonlinear function, because it has a square
  - $y = 0.25 + 0.5(x - 2)$ linear

TPACK Framework emphasised

Question 1: How is your understanding in linear functions enhanced by the teachers use of Geogebra

Response(S1); I am able to complete more work and stay focused throughout the lesson

Response (S2); I am able to see and observe what am doing so my interest is sustain throughout the learning

Response (S3): The use of GeoGebra assisted me to understand the work better because I can drag the graphs and understand what is happening

Response (S4): I am to follow what the teacher is teaching so I don't drift in my learning. This made me to follow and stay with the lesson. I like the pace of the lesson

Response (S5): I am able to practice more work and if am stuck I able to get help quickly and then I move on with my learning. The teacher is able to demonstrate and show me how to overcome my difficulties. The pace of the lesson is good and not boring.

Question 2: What do you like most when the teacher was teaching with GeoGebra?

Response (S1): using the software made my work so nice and neat, it was very user friendly since if I made a mistake, am able to erase my work and do it again. Something I cannot do when learning with pen and paper.

Response (S2): It saves time when you are using GeoGebra to do your class work.

Response (S3): The teacher is able to give clear instruction on what we have to do without sometimes talking too much

Response (S4):The teachers is able to walk around the class and support pupils having troubles and also provide extension work for the quick students.

Response (S5): The teacher is able to use different questions to access my learning ability on the class work.

Question 3: Would you recommend and be happy for the teacher to use GeoGebra to teach all the topics?

Response (S1): Yes, because I think the use of GeoGebra made the learning more fun and everyone in the class was paying attention to the teacher

Response (S2): learning with the software made the learning visual for me so I would like it to be used to present all topics in most lessons

Response (S3): Although I had some problems in starting with the work, my friends helped me to overcome my problems. I think it will be good to use it in all lessons to help children understand the learning better.

Response (S4): Using the software made the lesson very challenging for me. I was able to do most of the work alone by myself once I had been given clear instructions and demonstrations by the teacher. I think it will be ok for me for the software to be used always in my lessons.

Response(S5): I like to use GeoGebra in my all my lessons because I think teachers can give us homework and everyone will be able to do their homework at home if you have a computer since everyone like working on the computers.

Appendix 8: Letter to Head teacher

Dear Sir, I would like your permission to conduct a research project in the school for my dissertation as part of my PhD in Mathematics Education.

The title of the research: The impact of ICT on Mathematics education

The aim of the research: To explore my own classroom practice when teaching and learning linear functions using digital technologies. The focus of the research is on my interaction with the students' mathematical activity with digital technologies.

My year 9 maths group will be involved in the research study. By taking part in this research, the school will discover new approaches to the teaching of linear functions using digital technology. 10 students who have consistently demonstrated interest in integrated technology learning will be approached and provided with an information sheet explaining the purpose of this research. They will also be given a consent form for their parents to sign to show they are happy for their child to participate in the project. All students and parents will be informed about the expectations of the research.

The obligation expected from the school is to support me and permit me to use my year 9 maths group to take part in the study during one week starting in the first week of September, 2019. As part of the study, students will be recorded completing their class work in the maths lesson. All participants will be granted anonymity in the research reports. I will keep all the data collected strictly confidential and only make it available to the designated supervisor and myself.

If permission is granted to proceed with the project, I will send a letter to parents informing them about the project and their right to withdraw their child's data if they wish to.

Please contact me if you need further information about my study in the school.

Many thanks for your cooperation as usual and I look forward to hearing from you soon.

Yours sincerely,

D. Ata-Baah

Appendix 9: Letter to Teacher

1st September 2019

Dear Parent/Guardian

As part of my final module to complete my PHD Degree in Maths Education, I am undertaking a research project into how students learn linear functions using dynamic technology. I am delighted to inform you that your son/daughter is among six students selected to take part in a research project due to his/her interest in learning with technology and his/her articulated expressive reasoning in mathematics.

The research will take place in his/her normal teaching group and does not require any changes to his/her normal learning or timetable. During their normal lesson with me, I will observe how the pupils interact with technology when learning linear functions in mathematics. All participants will be invited to discuss their reasoning during the session with me at the end of the lesson. I will also ask them to complete a task that will require written responses.

Also after the session, students will be invited for a short interview to discuss their learning with the technology. Two lessons are planned, each lasting for about 60 minutes with 10 minutes of individual interviews during the lunch break. The interviews will not have any impact on their lunch break since pupils will be allocated extra time for their lunch.

The data collected from pupils' responses together with the response from the interviews will form the basis for the research. All pupils will remain anonymous and your son's/daughter's details will not be disclosed to a third party without your consent. All parents will have the right to withdraw their child's data from the project at any stage of the research.

This is an exciting opportunity for your child to take part in the research since I believe the experience will be enriching and worthwhile to develop pupils' deeper understanding of mathematics.

Please complete and sign the paper below to let me know whether you would like your son/daughter to participate in the research and return it to me by 30th September,2019. I will assume that you have consented to your child taking part in the research if I do not hear from you by then.

Many thanks.

D. Ata-Baah

Head and Teacher of Mathematics

Pupils Name---

I do agree… ☐

I do not agree that my son/daughter can participate and be interviewed ☐

(Please tick whichever applies)

Parent Name: -------------------------------------Signature: ---------------------Date: ----------

Appendix 10: Consent form

Title: What is the impact of ICT on mathematics education?

(Using GeoGebra to teaching linear functions)

01.09.2019-01.10.2019

Please complete this consent form and return to atabee@hotmail.com by 30.08.2019.

I am consenting to this element of the study by initialling/ ticking each box below Any unticked box will be assumed that I DO NOT consent to that part of the study. I understand that by not giving consent for any one element that I may be deemed ineligible for the study.

I agree to take part in this research,	Yes	No
I am free to withdraw at any point without giving a reason		
I agree that my name will be protected in the publication of any report		
I am happy for my interview to be audio		
I agree that the data will be collected and processed in accordance with the University's Data Protection Policy.		

Name ________________________

Signed ________________________ Date _____________________

Researcher's name: Daniel Ata-Baah

Signed __________

Appendix 11: Participation Consent Form

Title of Research Project:

What is the impact of integrating ICT on mathematics education?

Brief Description of Research Project,

The purpose of the project is to find out the impact of teachers on the use of ICT in teaching and learning mathematics. This research is carried out in fulfilment of my PhD degree in Mathematics education dissertation.

What Participation Involves.

10 participants will be selected for this project. I will conduct a semi structured interviews over a period of two weeks and then use about a month to analyse my data. The findings will be presented in a report. Each interview will last for about 15 minutes.

Limits on confidentiality

The interview will be tape recorded, and transcribed with any identifying details removed. The transcript, or extracts from, may appear in my report and in publications arising from it. The tapes may be heard by my supervisor and others who might be involved in examining the report.

Everything you say will be treated confidentiality, but there is a limit to this: if you disclose a risk of serious harm then I may need to take appropriate action (this adheres to the ethical guidelines of the ………..).

Student Investigator Contact Details:

Name: Daniel Ata-Baah,

Email: Telephone

Consent Statement:

I agree to take part in this research, and am aware that I am free to withdraw at any point without giving a reason, although if I do so I understand that my data might still be used in a collated form. I understand that the information I provide will be treated in confidence by the investigator and that my identity will be protected in the publication of any findings, and that data will be collected and processed in accordance with the Data Protection Act 1998 and with the University's Data Protection Policy.

Name ………………………………… Signature ……………………………… Date -
…………………………………

Please note: if you have a concern about any aspect of your participation or any other queries please raise this with the investigator or the Supervisor. However, if you would like to contact an independent party please contact the Head of Department.

Supervisor Contact Details:	**Head of Department Contact Details:**
Name:	Name
University Address:	University Address:
Email:	Email:
Telephone:	Telephone:

Appendix 12: Participant Debrief

Participant Number: __________

<u>Title of Research Project:</u>

What is the impact of ICT on Mathematics education?

Thank you very much for taking part in our study, we greatly appreciate your contribution.

This study is designed to examine the factors that might improve the integration of ICT by subject teacher and your participation is extremely valuable.

All data gathered during this study will be held securely and anonymously. If you wish to withdraw from the study, contact us with your participant number (above) and your information will be deleted from our files.

Should you have any concern about any aspect of your participation in this study, please raise it with the investigator in the first instance or with the Project Supervisor or Head of Department.

Investigator	**Project Supervisor**	**Head of Department**

Daniel Ata-Baah Name Name

If you are a student at Roehampton University and are troubled or worried about any aspect of the study, or issues it may have raised, you may find it helpful to contact one of the following who will be able to advise you on agencies that can deal with your particular concern:

Student Welfare Officers:

…………….

…………….

…………….

If you feel your concerns are more serious or complex you may wish to contact your General practitioner.

If you are a non-student you may find it helpful to contact your GP or one of the following who will be able to advise you on agencies that can deal with your particular concern:

[Agree with supervisor on appropriate agencies]

Appendix 13: Health & Safety Risk Assessment

Health & Safety Risk Assessment for Ethics Applications

RISK ASSESSMENT FORM			
Risk Assessment Reference Number			
Title of Project	**What is the impact of ICT in mathematics education?**		
Description of activity	• Interviews • Recording of interviews	**Date**	From: September 2020 Until:,
Area/Locations	London		

<table>
<tr><td rowspan="3">Project team</td><td>Name</td><td colspan="2">Job Title</td><td>Signed</td></tr>
<tr><td></td><td colspan="2"></td><td></td></tr>
<tr><td></td><td colspan="2"></td><td></td></tr>
<tr><td>Hazards</td><td colspan="4">1.None
2.
3.
Etc.</td></tr>
<tr><td>Who can be harmed?</td><td colspan="4">1. None
2.
Etc.</td></tr>
<tr><td>How can someone be harmed?</td><td colspan="4">1. None
2.
Etc.</td></tr>
<tr><td rowspan="2">Number of people affected</td><td rowspan="2"></td><td>Rate</td><td colspan="2" rowspan="2">H=Hourly, D=Daily, W=Weekly, M=Monthly, Q=Quarterly, S=Six monthly, A=Annually</td></tr>
<tr><td></td></tr>
<tr><td>Consequence</td><td colspan="4">1. None
2.
.Etc.</td></tr>
<tr><td>Existing Control Measures</td><td colspan="4">1. None
2.
Etc.</td></tr>
</table>

Comments						
Risk rating		**VH=Very High, H=High, M=Medium, L=Low, VL=Very Low**				
Further possible control measures	None					
Any further actions required						
Responsible person	**Description of hazard**	**Details of action taken**	**Date**	**Completed**		
The signatures below confirms that a meeting/discussion has taken place, the Hazard, Risks and appropriate Control measures outlined above have been read and understood.						
Signed (Applicant)		**Print Name**	Daniel Ata-Baah	**Date**		
Student Applications **Signed (Supervisor/PI)**		**Print Name**		**Date**		
Staff Applications **Signed (Peer Review)**		**Print Name**		**Date**		

Appendix 14: Interviews and Documentary Analysis Schedules

Dissertation title: What is the impact of ICT on Mathematics education?

Key Questions	Prompts and Probes
<u>Part 1: Beliefs on ICT</u> 1. What do you believe to be the benefits of using ICT in your subject area? 2. What do you believe are the Obstacles/barriers to integrating ICT into your Mathematics pedagogy?	Is the use of ICT integration a necessity in your mathematics? Why do you say that?
<u>Part 2 ICT Facilitators</u> 3. What do you perceive are the facilitators to integrating ICT into your department's pedagogy? 4. What are the specific tools used to facilitate ICT integration in mathematics?	How do you think the use of ICT can be supported the teaching of mathematics ? What Strategies for using ICTs would change pedagogical practices.
<u>Part 3: The impact of ICT</u> 5. How does your experience of using ICT impact the use of ICT in your teaching? 6. What is the impact of ICT on learning and achievement ?	Comment on your personal ICT experience in your pedagogy? Can you mention your current implementations of ICT tools in your teaching? Do you see any improvement in pupils learning using ICT?
<u>Part 4: Support to use ICT</u> 7. What on-going professional development is available to support the use of ICT? 8. What do you perceive to be the effective strategies for using ICT to change pedagogical practices?	What do you think are the barriers to ICT integration in your mathematics?

<u>Part 5: Leadership role on integrated ICT</u> 9: How do you view whole school leadership on ICT integrated learning? 10: How would you view your leadership role on integrated ICT pedagogy in your department?	Do you think the leadership in your school support the use of ICT in teaching mathematics? How do you feel about the leadership within your department on ICT integration? Are teachers supportive?

Appendix 15: Interview Transcript 1 – Teacher 1

00:00-00:14

Interviewer: This interview is in fulfilment of my Phd degree programme.

The tile of the research is 'What are the impact on ICT in Mathematics education?

00:15-00:20

Interviewer: You can always pull out of the interview if you don't feel comfortable.

00:21-00:21

Respondent: Ok.

00:21–00:27

Interviewer: Any information that I gather here will not be used externally without your permission.

00:2–00:28

Respondent: Ok.

00:28–00:56

Interviewer: Question: what do you believe to be the benefits of using ICT in in maths?

00:59–01:55

Respondent: I think that it brings life into the lesson. The development of concepts can be quite boring for some students, especially when it comes to reading and writing. I think, especially the internet has a lot of potential for young people to be engaged in different ways in the classroom and also it makes the learning more fun…

01:56–02:09

Interviewer: Question 2 Alright, very good. Now what do you believe are the obstacles, or the barriers, the problems to integrating ICT into maths pedagogy?

02:12–03:09

Respondent: I suppose that it would always depend on the flexibility of the curriculum and how the curriculum can be approached because, well, there are certain topics and certain concepts that have to be covered and, I think that maybe for the sake of following the curriculum it is impossible to integrate ICT. Also sometimes there is lack of resources. It will be great if the school can provide us the resources that we wasting time looking for resources.

03:09–03:03

Interviewer: Is there any more problems that you can think of?

03:09–03:11

Respondent: Yeah, exactly, the Hardware and software problem makes it difficult to use ICT with poor internet problems. It is very frustrating using the internet with slow internet connections.

04:12–04:27

Interviewer: Ok, now, question 3: what do you believe or perceive are the facilitators to integrating ICT into maths pedagogy?

04:31–05:30

Respondent: I think that the best way to facilitate ICT integration would be for the teachers to be ICT literate. I think that most teachers ultimately do not have the knowledge; so on the job training will help to facilitate ICT integration.

05:30–05:34

Interviewer: How does your experience in using ICT impact on your ICT use in your teaching? Your … I'm talking about your own personal experience in using ICT.

05:59–06:30

Respondent: Well, when it comes to teaching, in particular, I think that there are some videos, films, adaptations of books that sometimes help a lot to the children to understand the background or put into context all the things that we imagine when we're reading, so I think for me, II think my good experience in using ICT has helped me to integrate ICT tools into my lessons. I like to use ICT to promote visual learning through the interactive white boards.

06:30–06:32

Interviewer: Ok, good. Question what is the impact of ICT on learning and achievement? Do you think the use of ICT brings about higher achievement in learning?

08:22–09:28

Respondent: I couldn't say for sure because I think that it makes a great contribution into engaging students better and I think that the interactive nature of ICT tools makes it more enjoyable and promotes independence.

09:28–09:39

Interviewer: Ok, question: what ongoing professional development that you know is available to support you to use of ICT?

09:44–10:20

Respondent: Well, at school we have ICT person that is in charge of smart boards and computers and we have, online safety course organised by external agencies.

10:20–10:20

Interviewer

What do you believe to be the support on ICT integrating in your school?

12: 00-12:30

There is no support for staff. Because guess when you are struggling and want some help, you will find it difficult to get the needed training since most of the IT personals may be working on other projects for the school or sometimes supporting children with passwords and trivial IT issues with teachers such as the white board is not working, classroom speakers are not working and so not any major training takes place. There is no external support as well and I can say that it is a struggle with most teachers.

Respondent: I think that there should be a strong school policy on in-service teacher training initiatives to ensure that ICT is integrated in every lesson. I mean unfortunately for teachers as well as leaders, this is something very recurrent I think. It's like the training never stops - we're always trying to catch up with something that we need to be, you know, up to date with. So I would say that training is everything… is key to incorporating this, I know other strategies will include changing the culture of the school by giving incentives to teachers who regularly integrate ICT in their lessons will help.

14:04–14:09

Interviewer: Are there any habits or attitudes or perceptions that need to be changed?

14:07–15:04

Respondent: Yeah. Yeah … yeah … yeah, and also there are so many constraints and so many pressures, from … from … from a budget perspective, for the coming years. I don't know what is going to happen with there are Ofsted visits, I mean schools that … that need to … that are, you know, that have that cloud above their head waiting for the call and praying for a Wednesday to cancel they didn't receive the call and they're now being Ofsted and there is a lot of pressure there and I think that there is such a focus on achievement and on league tables and being at the top and advancing x number of levels per year and we lose that creative part, that … the … the freedom to … to try to explore in different ways of teaching that could be more enjoyable but they are maybe less accountable for …

15:04–15:24

Interviewer: That's very good, very good. Ok how do you perceive ICT integrated learning in your lessons? Do you think it's very necessary to promote it, is it a necessity, do you think it makes no difference or …?

16:59–17:24

Interviewer: Now, how do you view your leadership role for … on integrating ICT pedagogy in your department?

17:24–18:54

Respondent: I suppose that there's a lot to be said about example setting and examples. I think the leader should lead by example. is being an example and being a team player as well, so trying to bring out the best of everyone involved, so I think that the fact that I am a very ICT person … I use apps for pretty much everything in my daily life … makes it more believable when I say like, well, we can use this to enhance that or this activity we could incorporate the tablet for this or dictionaries for that or whatever. I think that in this particular case, if you're not an example, then you're not a true leader. Yeah.

18:54–18:56

Interviewer: Alright. Thank you very much, is there anything you would like to add?

Appendix 16: Interview Transcript 2 – Teacher 2

00:00–00:31

Interviewer: These are questions for my Phd degree in using ICT in mathematics

Education.You are free to pull out of this interview at any point in time No information from you will be given to a third party without your permission.

Ok, so what do you believe to be the benefits of using ICT in teaching maths?

00:32–00:54
Respondent: It's interactive side, mainly. I'm as a teacher, I use a lot of ICT to teach maths, purely because its interactive for the young children, you can make it as animated as possible for the children, in particular. You can change a lot of things, so it's not like books and stuff where every-, all the information is static, its actually things you can change when and if you want to, so it's really, really flexible.

00:56 – 01:04

Interviewer: Alright, what do you believe are the obstacles or barriers to integrating ICT in teaching maths?

01:05–01:18
Respondent: Yeah, I think cost is one of the ones, secondly its reliability. You know if the system fails or internet fails or power fails, then you have those issues. So, yeah, my main two ones would be … would be cost and reliability.

01:19 – 01:29

Interviewer: Right, well, what do you believe or perceive as the facilitators to integrating ICT in teaching maths?

01:37–02:00
Respondent: Yeah, I think generally make it more widespread. I know my school is using it a lot more and, so I think if everyone gets on the same platform and understands the benefits of using this, it would really help all the the children. So I think just making sure everyone's aware why we use ICT, and the benefits of it, and once everyone understands that it'll be a lot … lot clearer to everyone to be supported.

02:00 – 02:17

Interviewer: Ok, so do you think your expertise or knowledge in ICT would really be a facilitator in you helping to use ICT in your lessons?

02:17–02:43

Respondent: Yeah, of course, I think … I think everyone one needs to have a certain degree of ICT skills to be able to teach the children. I think with ICT being advanced in so many ways, you know using a computer or using a tablet, you know everyone has to have a certain degree of skills, so I think having that basic knowledge at least in ICT you can then help children develop their skills. And with ICT you're always learning, there's always things like upgrades that come along and … and your various development. I must inform you that am not a fun of training provided by external consultants. We have not been lucky with this due to lack of follow ups.

02:44 – 02:50

Interviewer: Good. Thank you. How does your experience, lets say your background impact the use of ICT?

02:50–03:14

Respondent: Yeah so my interest in ICT in the past is crucial, I think my past good experience helps to use ICT. Of cause my background as an IT consultant has made me appreciate the need to use ICT in most of teaching. This motivates me to plan, teach and always find ways to incorporate ICT.

04:47 – 04:55

Interviewer: Ok, is there any professional development that you think you would … you'll benefit from, to help you to use ICT?

04:58–05:23

Respondent: Yeah, I mean I myself attend a lot of trainings which involve ICT and to do with my maths teaching and I think that will benefit everyone, I think for all, you know, children are using computers from a very young age and some adults are barely using computers. So, I think it's very important that they also keep up with … with technology to then keep up with their children and it will help their teaching aspect of the children as well.

05:54 – 06:08

Interviewer: Ok, now, how do you view the role of ICT integrated leadership on learning? Is it very, very important, in your view to integrate ICT …? Is it a necessity or can … is it like a luxury? Can the school just do without it or does it make any difference at all?

06:08–06:18

Respondent: I think it's a necessity to have a strong leadership on ICT in the school and I think it's a great tool for school to improve student achievements with a strong leader who will develop the entire school to integrate ICT.

06:18 – 06:35

Interviewer: Ok. Thank you, the last question now. How would you view your leadership role on integrated ICT pedagogy in the maths department? I mean your leadership skills, how do you think will impact the teachers in the department to be encouraged to use ICT?

06:35–06:56

Respondent: Yeah, I think with, you know, the background knowledge of how ICT can help and my background skills in IT, I continue to show the other teachers how to integrate ICT through training meetings, you know such training will boost teachers confidence. You can't just say ICT is going to help, you need evidence, you need proof and with that you have a stronger argument, so I think having that base knowledge, it's going to put teachers in the department in a stronger position to use ICT.

Interviewer: Thank you. The outcome of this will be publicised, so in case you're interested in it, I can let you have a copy of the research.

07:07–07:08

Respondent: Yeah, alright, yep.

Appendix17: Interview Transcript 3 _Teacher 3

00:00–00:03

Respondent: I can't guarantee that I'll be able to answer your questions, but … I will try.

00:04–00:08

Interviewer: Please try your very best. These questions are to assist me complete my dissertation in the Phd degree in Mathematics Education. Please feel free to let me know when you want to pull out of this interview at any stage. No information from you will be given to a third party without your permission.

00:09 – 00:18

Interviewer: Ok, so, what do you believe to be the benefits of using ICT in mathematics?

00:19–00:40
Respondent: It connects well to the majority of student's preferring to learning using ICT due to the interactivity and instant feedback to class work…

00:48 – 01:04

Interviewer: Good. What do you believe are the obstacles then for … integrating ICT in your pedagogy?

01:04–01:13
Respondent: Yeah, I don't see any obstacles. The only obstacle is when it's not working …

01:13 – 01:17

Interviewer: You mean the hardware isn't working or is it the issues with the software?

01:17–01:54
Respondent: Yeah, when you think about the work to be done and the poor internet connection is not functioning or there's been a, some kind of internet problem, yeah, obviously there are problems with monitoring children's work due to lack of the software to monitoring them. Children are carried away by doing other task or playing games on the computer.

01:54 – 02:10

Interviewer: But in terms of the software or, lets say, a teachers ability, if the teacher hasn't got the expertise in using ICT, don't you see it as an obstacle?

02:10–02:19
Respondent: Yes, that's an obstacle as well, yeah. Yes, there is quite a problem with expertise, yeah. So that is yes.

02:19 – 02:29

Interviewer: But in terms of the curriculum, what do you think? Doesn’t it affect the way the curriculum is delivered?

02:29–03:10

Respondent: No, no it doesn’t, it actually does the opposite. Not only that, but when it comes to real-life examples you can show video clips like ‘science in the real world’, just to … just to help the students connect better than …, because they can see its application in the real world.

05:55 – 06:11

Interviewer: Ok, so, do you think a good leadership on ICT is really a necessity in raising people’s achievement and how do your think teachers can be supported to integrate ICT?

06:12–07:25

Respondent: As I’ve said before it’s really down to personal preference. Preference but, I guess it’s to do with preference and as I said, the joys of the leader in using ICT. An interactive whiteboard in itself is, you know, it’s an interactive activity that a download going on between yourself and the students. There can be good at learning using ICT interactive tools with a teacher who knows what he is doing.

09:56 – 09:14

Respondent: Training sessions help such teachers to overcome their difficulties. Yeah, that’s one thing that needs to be acknowledged than to discourage them from using ICT. Although it’s very rare that teachers haven’t got the experience of ICT that can’t be used. For example, using the spreadsheet or to analyse data or to input test scores onto the computer. These are simple tasks that every teacher can do. The more completed task should be learned through training sessions provided by the subject leaders if they know or ICT expects can be invited to train all teachers who are lacking the required skills needed for teaching science.

10:49–10:55

Interviewer: But have schools really got the time… got those training programmes in place, to help subject teacher?

10:55 – 11:25

Respondent: I think that maybe there are schools that don't really know where to begin and they kind of, lack motivation, so that is schools coming together and working together. Our school sometimes tap into experienced teacher's expertise and ask them to offer insets and share good practice.

11:25–11:3
Interviewer: But the problem with schools is that there's no follow-up, so teachers don't really benefit from those insets or what do you think the impact of these training sessions are?

11:36 – 12:20

Respondent: Yeah, the follow-up is on time, yeah. It can be a problem. The follow-up can be a problem. And sometimes that's why working in teams or pairs does help us teachers. Working in pairs does help, like sometimes, I feel like I … I might know how to think about how the lesson will go and then my colleagues however, there might be a wizard on a computer, so we will work together all the time and then we'll get together and work on something substantial together, you know.

Appendix 18: Interview Transcript 4 – Teacher 4

00:01–00:10
Interviewer: Good morning. These questions are to assist me complete my dissertation on a Phd degree in Mathematics Education. Please feel free to let me know when you want to pull out of this interview at any stage. Also be informed that no information from this interview will be given to a third party without your permission.

Interviewer: What do you believe to be the benefits of using ICT, in teaching mathematics?

00:13 – 00:49

Respondent: Well, personally I believe the benefits of integrating ICT is uncountable. It makes my planning of lessons easier, I can teach using power points and online resources. I can make my lessons more interactive and by using ICT tools am able to plan and teach lessons to meet all the different learning styles in my class. Through ICT new skills and concepts are introduced. Also, when using ICT, I am able to promote more interest in the learning and this helps me with the classroom management and reinforces the learning.

Respondent: And because of that, they will have studied better, they have working knowledge of what they are doing, and they, it will increase their attainment.

01:23 – 01:30

Interviewer: Okay. So do you think without ICT, the people's attainments cannot be increased?

01:31 – 01:52

Respondent: Well, without ICT guess what I could to as a teacher? I will not be able to assess and obtain instant feedback from completed student work. You know learning can be very boring. ICT gives me the chance to differentiate my teaching and so it is a good tool to raise attainment of most children at the same time. Raising attainment would not be rapid without the use of ICT tools in lessons.

02:03-02:10

Interviewer: Ok my next question. What do you believe are the obstacles for integrating ICT as a subject leader of Mathematics in your school?

02:13 – 02:14

Respondent: The software. In my experience the barriers include insufficient number of computers, insufficient number of internet-connected computers, Insufficient Internet bandwidth or speed, sometimes and the lack of adequate skills of personals to support teachers. Also there is the restriction on the school's timetable. The time is fixed with lesson time and the inability to find time for training is a concern for most teachers.

03:20 – 03:25

Interviewer: What do you perceive are the facilitators to integrating ICT into your department's pedagogy?

03:25 – 03:40

Respondent: I think among the supporters or drive of ICT in my school will be the personal or any person who is more experienced and knowledgeable to drive the initiative forward. The technology co-ordinator is able to design programs to drive the agenda. The school can employ people externally to drive the initiative. This can be done through online coaching services.

03:40 – 03:50

Interviewer: what about teachers' experience? How does your experience of using ICT impact the use of ICT in your school?

04:30 – 04:53

Respondent: I have had difficult times using ICT due to lack of sufficient knowledge in ICT from the teacher training school that I attended. Less ICT was used when learning maths so I find it more comfortable just sticking to what I already know. In my school, using ICT in teaching and learning has not being a priority. I guess as long as the children achieve their targets, the school was happy with you. For me as a leader of maths, I am also happy as long as the department does well.

04:53 – 05:59

Interviewer: So, teachers who are not confident… Is there any program to support them?

05:00 – 05:05

Respondent: Sometimes we do have a program in the department to support teachers. For that we do, but there is the lack of time to learn and gain the required confidence. Occasionally we train teachers to use new software properly.

Interviewer: How do you view whole school leadership on ICT integrated learning?

05:30 – 06:40

Respondent: The priority of the school is to perform well against schools in the catchment area so I guess integrating ICT is not a big drive in the school. I may be wrong, but often the focus is on outstanding lessons and it does not matter how a teacher does it. Usually, at the start of the term, there are series of training including using ICT across the curriculum. However, you don't hear anything about it again during the term and there is no clear direction on how the focus on ICT is champion in the school.

06:30 – 06:50

Interviewer: How would you view your leadership role on integrated ICT pedagogy in your department?

06:50 – 07:20

Respondent: I have the passion to integrate ICT but I don't have the skills and confident to provide the leadership drive on it. I have promoted department trainings to motivate the team in the department. However, lack of consistencies with the whole school policy on ICT makes it difficult to measure the outcomes of ICT use.

07:42-07:45

Interviewer: Thank you and that's the end of the interview.

Appendix 19: Interview Transcript 5 – Teacher 5

00:01–00:15

Interviewer (A): Hi. These are interview questions to aid me write a dissertation, in partial fulfilment of my PhD degree in mathematics Education. You may feel free to pull out of this interview at any stage if you are not comfortable with the questions. I can assure you that no information from this interview will be given to a third party without your permission.

00:15–00:25

A: What do you – What do you believe to be the benefit of using ICT in your Mathematics education?

00:25–00:51

B: OK, I would say the benefit; the main benefit of using ICT is that it offers a greater degree of interaction with the students. It offers a greater range of interaction than a conventional whiteboard and black pen. I would say to you that it offers a greater variety and, if you like, breadths and depths of interaction.

02:23–02:31

A: What are the obstacles or barriers to integrating ICT?

02:31–03:06

B: Oh, great! So, you can put that—you can say one of the obstacles, the barriers to ICT, is even though an interactive whiteboard has many functions, it's not personalized enough. One of the barriers is there should be greater use of radio-controlled pads, so that—or linked notebooks for the children so that they can directly interact with the teacher at the front. Also, the time and space available on the schools time table and the lack of interest of the school leaders are creating barrier to the ICT integration in the secondary schools' pedagogy.

03:24–03:52

B: No, I mean the national curriculum could facilitate—I mean you could use ICT in *any* Mathematics lesson. For example, if you're teaching geometry, yeah? If you're introducing, you know, straight line theorems, intersecting line theorems at Key Stage 3— you can use, you can use Geo—GeoGebra what are they called?

06:56–07:10

A: Yeah, but with this, what would you think about the teachers and personal experience? If a teacher isn't so confident in using ICT, do you think that would promote ICT integration?

07:50–08:08

A: But as a subject leader, if you are not confident with ICT, are you sure that your leadership on ICT in the department would be beneficial to other members of staff?

08:09–09:32

B: Yeah…I think as a head of department, you don't necessarily have to be the best person on every piece of software in the department. Yeah? I think it helps if you have those skills, but if you don't have those skills, then I would have champions within my department. So, for instance, if you had, Daniel, I don't know, me in your department, and I was hot on Cabri, you'd let me run the training and take charge of training people within the department. You know? And I think sometimes it doesn't happen because people are – we are too ambitious sometimes, and what we should do is maybe start off with, like, planning a small part of the curriculum using ICT, and developing those materials and storing it, rather than trying to do the whole lot. We have to start small and then build big. But I think within – that's where leadership of learning is important. Because as a leader, you're not the person that necessarily does execute, but you're the person that makes sure those things get done.

09:33–09:39

A: But if you're a leader who doesn't believe in…you see, there are some leaders who don't believe in ICT, so would they be able to provide leadership on something they don't believe in?

09:40

B: Leading doesn't mean that you do everything.

09:43–09:45

A: You have to believe in it before you use it. Isn't?

09:47–10:52

B: I'm saying good leadership means you've got to distribute the leadership within your team. Now, you know, if you want somebody to be an expert in Geometer's Sketchpad, you could give them time to do some training, give them time to do… to sort of send them on a course, do you understand what I'm saying? And then they have that time back in here to pay it back somehow by taking part of a departmental meeting, by rolling out some new materials well in advance, getting some feedback from the kids and from the students about how it went. There's no point doing it if it's not working. The whole point of doing it, of implementing the change, is that it's going manifest itself in the children's understanding, isn't it? So, you know it's – but I do agree with you that being a good role model is good, but I know guys who are good themselves, but nobody in the department ever can keep up with them, and it doesn't happen. And I've seen situations where people have been in the department, and been more distributive leaders, and it's been far more effective.

10:53–11:09

A: But here is a case where the leader hasn't got any motivation to use ICT. So do you think he would promote any lead at all on teachers using ICT? He wouldn't.

11:10—11:15

B: No, he wouldn't, but that would be a problem for Senior Management, wouldn't it?

11:15–11:26

A: So it takes a leader who has got the charisma or who are interested in promoting ICT to really promote ICT in the department, otherwise ICT will be dormant. Is that right?

11:26–12:00

B: Basically, ICT is a tool. Just like pens, it's a tool. It's not a god, it's a tool. And I would say anybody who's interested in the teaching of maths to children has got to being using these tools. I mean, look at all of us, we're using mobile phones now. You go to the – you can't even pay your bus fare. If you've got money, you can't even go on the bus. We're using technology more and more in society, and I think they're in danger, if we don't use it in the classroom, we're going to actually lose the children.

14:52–15:17

A: That is very good. So those departments which haven't got leaders, who are so much into ICT, are there any professional developments which are taking place, in your opinion, which are preparing leaders to be able to integrate ICT in their lessons? Do you know of any professional developments happening around schools?

15:18–16:01

B: Oh, yeah, I mean in schools, I mean really there was a little bit of a wave but it seems to have died a death, and one of the reasons why it's died a death is because there isn't the money around. But I think, ideally, new schools are doing a lot of ICT training because they're buying products that are new, aren't they? They're buying products that are new. So sometimes when you go to a school, if you get a job in the school – and this is something you need to think about when you move schools – it's good to move to a school which just introduced some ICT. Because it means that nobody knows how to use it, and you will get a certain amount of time for training.

16:02–16:17

A: But wouldn't you just say that subject leaders who have had poor training from using ICT, example through PGCE or wherever, would have poor impact on the way they would use ICT in lessons?

16:17–17:36

B: Oh, yeah, obviously, if – I'd be very surprised if at PGC E they hadn't talked about, on a mass education course, they hadn't talked about the use of ICT. In the 21st century. I'd be very surprised. But I'm not saying that they all do. But I would have thought that on a teacher training course, they're going to discuss ideas like group work, individualized learning programs – which seems to be making a comeback. In the old days it was Smile but now it's MyMaths, they are making a comeback but they're kind of making a comeback in a supportive role. You know what I mean? They're used as homework. And that's ICT. I was at a school in Southwold today, and they actually had the MyMaths book. I'd never seen that before. MyMaths actually produce a book. And what they've cleverly done, they've matched their chapters to the topics they already have. So the idea is, if you buy into MyMaths, the book, you get exactly the same topics on the system.

17:59–18:23

A: Yeah, they've made lots of money. And lastly, how would you assess the impact of, let's say, local training, mentoring leaders who haven't got the ICT expertise? How would you assess the impact against schools who would be interested in bringing in external trainers?

18:24–21:37

B: I think one of the problems that there has been – and it's like anything – is having a coherent plan of implementation. We know that ICT has tremendous potential, but the problem that arises is how people, you know, how they implement the professional development, and do they actually review it, and then complete that cycle? Do you know what I mean? Reviewing the plans are the problem. People often introduce something, and they don't think carefully about how to monitor it. They just rush to introduce something. I mean, what I would say is, within the department, maybe, depends on how big the department, but you could start off with a champion, you know what I mean? Like, one person in the department who has some responsibility for that. And the way you could sell that to them as something that they could put on their CV. Yeah?

21:39–21:42

A: We have come to the end of the interview. Is there anything you would like to add?

21:43–22:45

B: I've given you a few ideas, isn't?

22:46–22:48

A: Alright. Yeah, that's fine, thank you.

Appendix 20: Teachers questionnaires

Objective:-

To collect information on the impact of using ICT in mathematics teaching in secondary schools in London, partial fulfilment of PhD Degree in Mathematics education.

Note:

- All answers will be kept confidential.
- You will be given the copy of the analysis if interested for your information.
- All teachers' identities would be protected.

Section A: Please tell us About Yourself.

Please tick one answer in each question.

Q1.Gender: *[]* MALE *[]* FEMALE

Q2. Age:--------------------------- **Q3 Position/teaching role: -----------------------------**

Under 30 years	30-40	Above 40

Q4.Teaching Experience

Under 5 years	*5 – 10 years*	*Above 10*

Q5. Educational Qualification

Diploma	Degree	Masters

Section B: use of ICT for learning mathematics.

Q6. How will you rate you ICT experience?

Below average	***Average***	***Above Average***

Q7. How often do you use ICT in teaching Mathematics?

Never	Sometimes	Always

Please tick one Strongly SA = Agreed , A = Agreed, D =Disagreed SD=Strongly Disagreed

Q8. What do you perceive to be the benefits of integrating ICT pedagogy in mathematics?

Sustained student interest.	Aid lesson planning with wide range of resources.	Improved the attainment of children.	Enhance interactivity and exploration.

SA	A	D	SD	SA	A	D	SD	SA	A	D	SD	SA	A	D	SD

Q9 What do you perceive to be the barriers of integrating ICT pedagogy in mathematics?

Lack of curriculum resources	Software and Hardware problems	Lack of experience with ICT tools	Negative attitudes.

SA	A	D	SD	SA	A	D	SD	SA	A	D	SD	SA	A	D	SD

Q10. What do you perceive to be the facilitators of integrating ICT pedagogy in mathematics?

The Availability of ICT tools	Experience and personal knowledge of using ICT	School leadership	Opportunities for training

SA	A	D	SD	SA	A	D	SD	SA	A	D	SD	SA	A	D	SD

Tables

Table 1 Mathematics teachers: interviews

Table 2 Mathematics teachers: Questionnaires

Table 3: Computer sharing

Table 4: Summary data analysis using SPSS